Microwave couplers

Outline

- Theoretical approach
- Ideal coupler
- Real coupler
- Applications :
  - Hybrid ring
  - Hybrid coupler

4-port scattering matrix

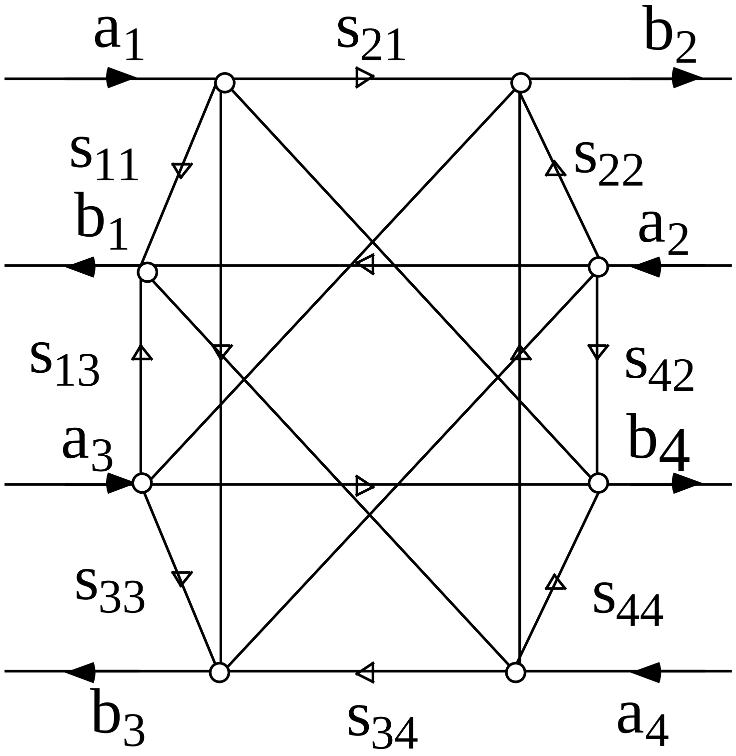
$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix}$$

General

$$\begin{bmatrix} 0 & s_{12} & s_{13} & s_{14} \\ s_{12} & 0 & s_{23} & s_{24} \\ s_{13} & s_{23} & 0 & s_{34} \\ s_{14} & s_{24} & s_{34} & 0 \end{bmatrix}$$

matched and reciprocal

flow chart



adapted and reciprocal and lossless 4-port

$$\begin{bmatrix} 0 & s_{12}^* & s_{13}^* & s_{14}^* \\ s_{12}^* & 0 & s_{23}^* & s_{24}^* \\ s_{13}^* & s_{23}^* & 0 & s_{34}^* \\ s_{14}^* & s_{24}^* & s_{34}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & s_{12} & s_{13} & s_{14} \\ s_{12} & 0 & s_{23} & s_{24} \\ s_{13} & s_{23} & 0 & s_{34} \\ s_{14} & s_{24} & s_{34} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

What does it mean ?

$$s_{13}^{*}s_{14} + s_{23}^{*}s_{24} = 0 \qquad (line\,3\,\mathrm{x}\,column\,4) \qquad \mathrm{x}\;s_{14}^{*}$$

$$s_{13}^{*}s_{23} + s_{14}^{*}s_{24} = 0 \qquad (line\,1\,\mathrm{x}\,column\,2) \qquad \mathrm{x}\;s_{23}^{*}$$

$-$

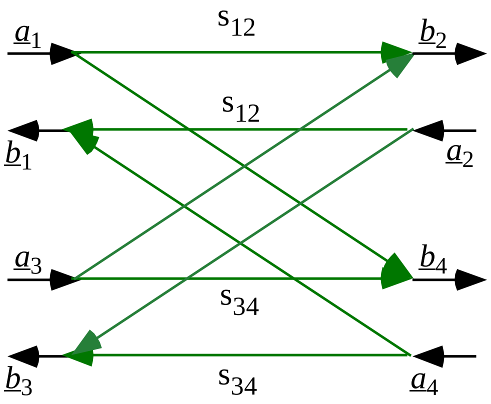
$$s_{13}^{*}\left(\left|s_{14}\right|^{2} - \left|s_{23}\right|^{2}\right) = 0$$

$$s_{13}^{*}\left(\left|s_{14}\right|^{2}-\left|s_{23}\right|^{2}\right)=0$$

1st solution : $\boxed{s_{13}=0\ ,\ s_{14}\neq 0\ \ ,\ s_{23}\neq 0}$

thus : $\boxed{s_{13}^{*}s_{14}+s_{23}^{*}s_{24}=0\ \Rightarrow\ s_{24}=0}$

$$\begin{bmatrix} 0 & s_{12} & 0 & s_{14} \\ s_{12} & 0 & s_{23} & 0 \\ 0 & s_{23} & 0 & s_{34} \\ s_{14} & 0 & s_{34} & 0 \end{bmatrix}$$



$$s_{13}^{*}\left(\left|s_{14}\right|^{2}-\left|s_{23}\right|^{2}\right)=0$$

2 nd solution : $\left|s_{14}\right|=\left|s_{23}\right|$

selection of reference planes : $s_{14}=s_{23}=j\beta$

$$\left|s_{12}\right|^{2}+\left|s_{13}\right|^{2}+\left|s_{14}\right|^{2}=1 \qquad (line\,1\,\mathrm{x}\,column\,1)$$

$$\left|s_{13}\right|^{2}+\left|s_{23}\right|^{2}+\left|s_{34}\right|^{2}=1 \qquad (line\,3\,\mathrm{x}\,column\,3)$$

thus : $s_{12}=s_{34}=\alpha$ with adequate ref. planes

2^{nd} solution

$$s_{12}^{*}s_{24}+s_{13}^{*}s_{34}=0=\alpha\left(s_{24}+s_{13}^{*}\right) \qquad (line\,1\,\text{x}\,column\,4)$$

$$s_{13}^{*}s_{14}+s_{23}^{*}s_{24}=0=\beta\left(s_{13}^{*}-s_{24}\right) \qquad (line\,3\,\text{x}\,column\,4)$$

$$\alpha=0\;,\;\beta=0$$

$$\begin{bmatrix} 0 & 0 & s_{13} & 0 \\ 0 & 0 & 0 & s_{24} \\ s_{13} & 0 & 0 & 0 \\ 0 & s_{24} & 0 & 0 \end{bmatrix}$$

$$s_{13}=s_{24}=0$$

$$\begin{bmatrix} 0 & s_{12} & 0 & s_{14} \\ s_{12} & 0 & s_{23} & 0 \\ 0 & s_{23} & 0 & s_{34} \\ s_{14} & 0 & s_{34} & 0 \end{bmatrix}$$

Ideal coupler

$$\begin{bmatrix} 0 & s_{12}^* & 0 & s_{14}^* \\ s_{12}^* & 0 & s_{23}^* & 0 \\ 0 & s_{23}^* & 0 & s_{34}^* \\ s_{14}^* & 0 & s_{34}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & s_{12} & 0 & s_{14} \\ s_{12} & 0 & s_{23} & 0 \\ 0 & s_{23} & 0 & s_{34} \\ s_{14} & 0 & s_{34} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|s_{12}|^2 + |s_{14}|^2 = 1$$

$$s_{12}s_{23}^* + s_{14}s_{34}^* = 0$$

$$|s_{12}|^2 + |s_{23}|^2 = 1$$

$$s_{12}s_{14}^* + s_{23}s_{34}^* = 0$$

$$|s_{23}|^2 + |s_{34}|^2 = 1$$

$$|s_{14}|^2 + |s_{34}|^2 = 1$$

Ideal coupler

$$|s_{12}| = |s_{34}| = \alpha$$

$$|s_{14}| = |s_{23}| = \beta$$

$$\alpha^2 + \beta^2 = 1$$

$$s_{12} = \alpha\, \mathrm{e}^{j\varphi}$$

$$s_{34} = \alpha\, \mathrm{e}^{j\eta}$$

$$s_{14} = \beta\, \mathrm{e}^{j\psi}$$

$$s_{23} = \beta\, \mathrm{e}^{j\theta}$$

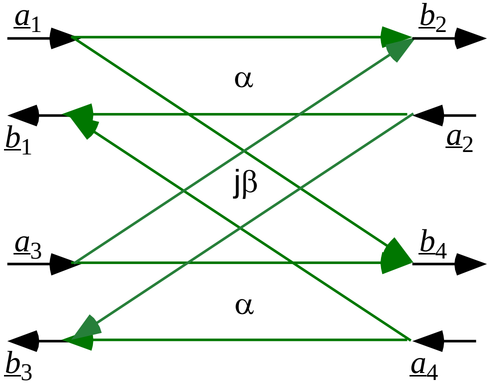
with $$(\varphi + \eta) = \pi + (\psi + \theta) + 2n\pi$$

and $\alpha > \beta$ by convention

symmetric coupler

$$\psi = \theta = \pi/2$$

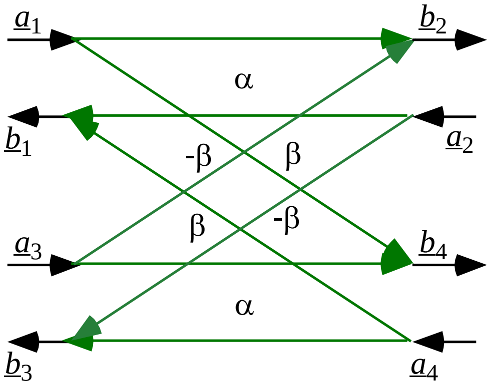
$$\begin{bmatrix} 0 & \alpha & 0 & j\beta \\ \alpha & 0 & j\beta & 0 \\ 0 & j\beta & 0 & \alpha \\ j\beta & 0 & \alpha & 0 \end{bmatrix}$$



asymmetric coupler

$$\psi = 0\ ,\ \theta = \pi$$

$$\begin{bmatrix} 0 & \alpha & 0 & \beta \\ \alpha & 0 & -\beta & 0 \\ 0 & -\beta & 0 & \alpha \\ \beta & 0 & \alpha & 0 \end{bmatrix}$$

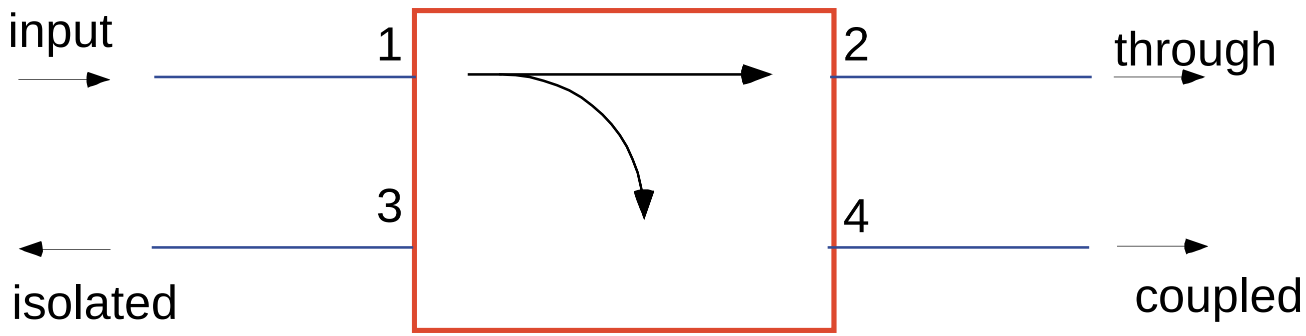


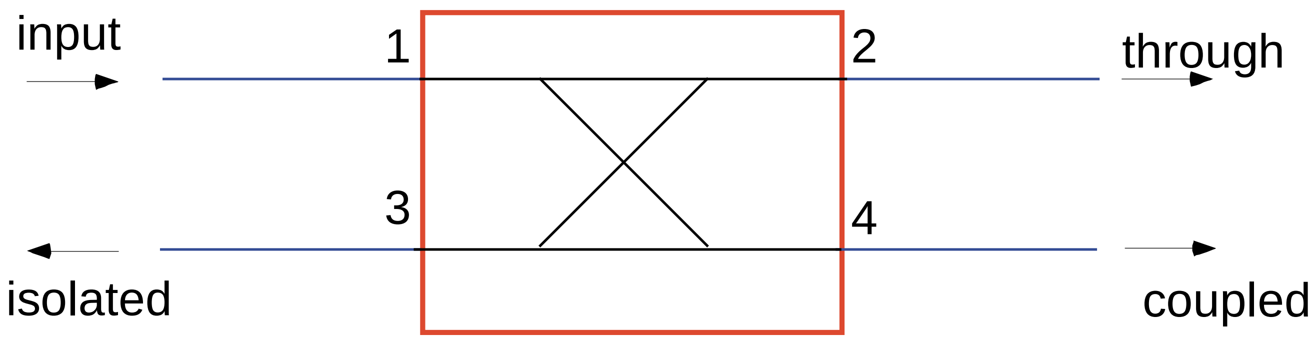
Summary : ideal coupler

Coupling : LC = -20log10 β

Attenuation : LA = -20log10 α

$\alpha \geq \beta$



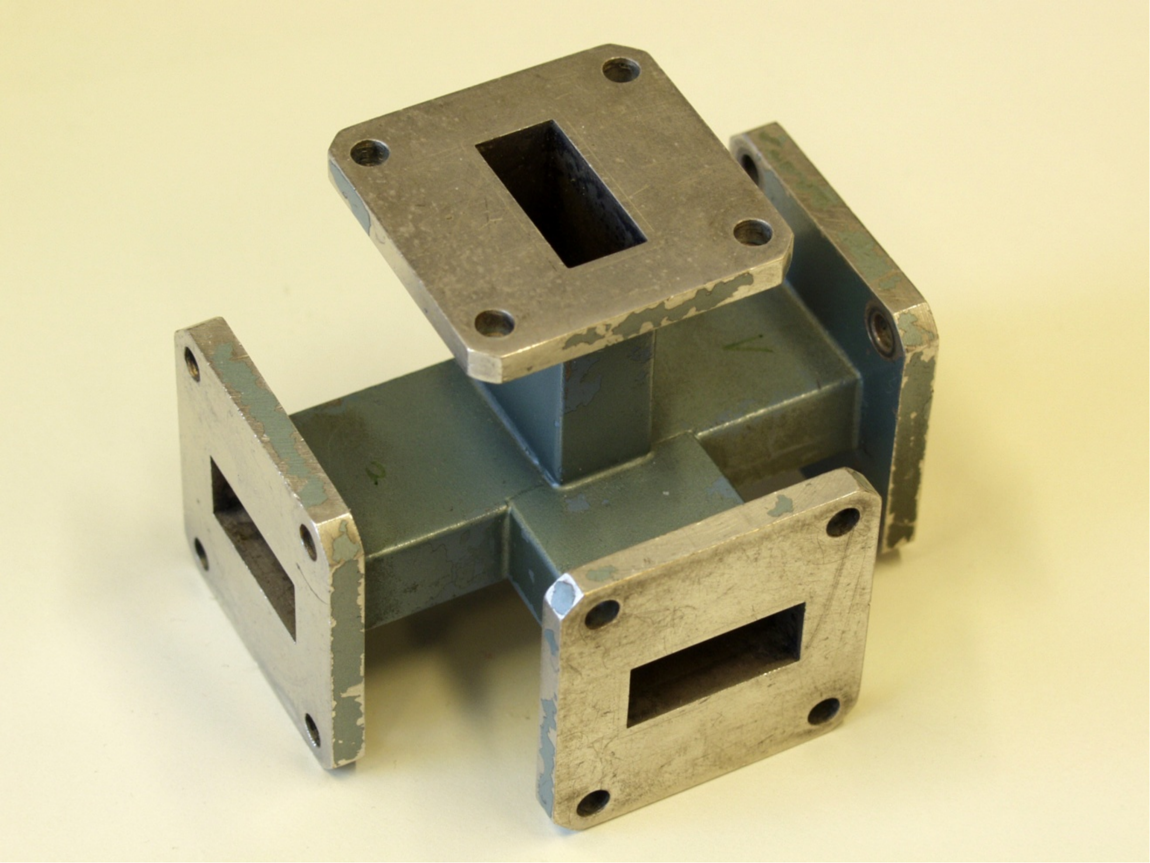


real coupler

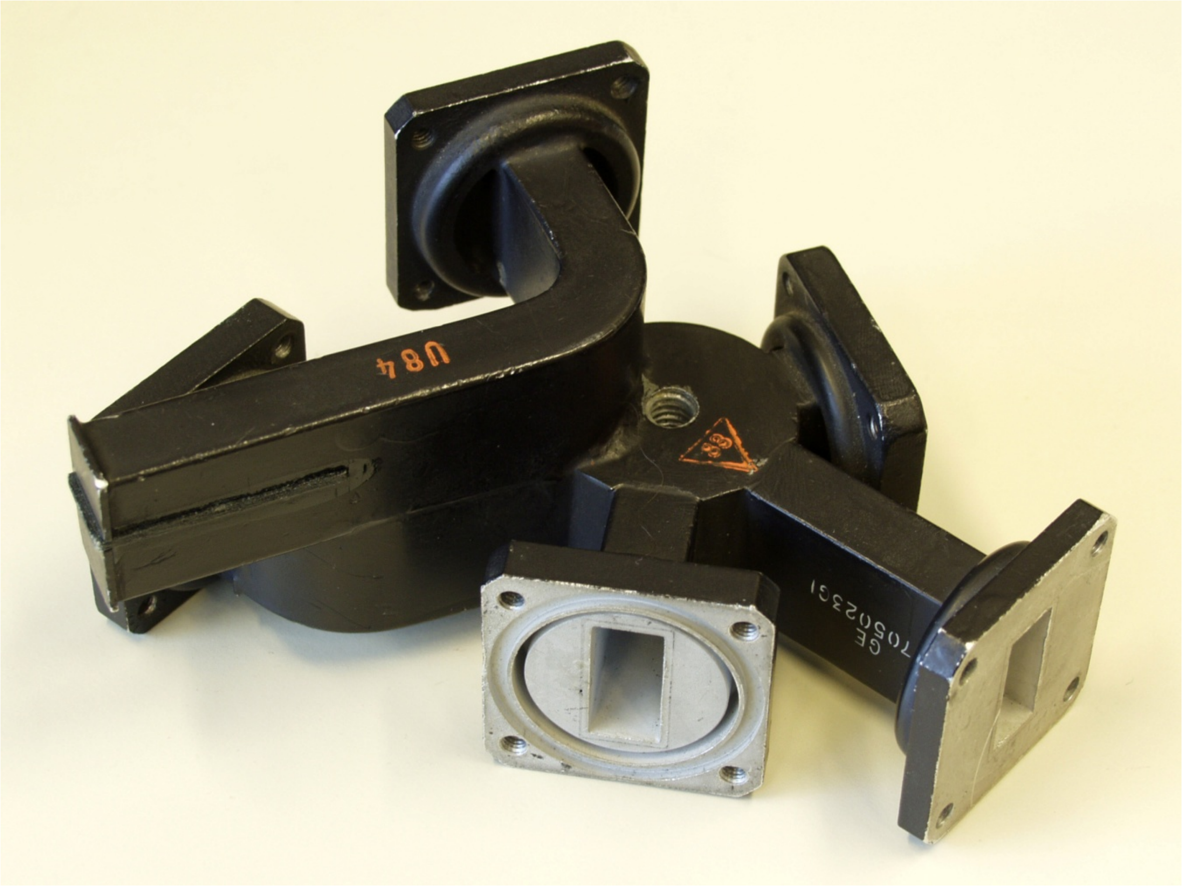
- $\alpha \geq \beta$
- Attenuation : $LA = -20\log_{10} \alpha$
- Coupling : $LC = -20\log_{10} \beta$
- Isolation : $LI_{31} = -20\log_{10} |s_{31}|$
- Isolation : $LI_{24} = -20\log_{10}|s_{24}|$
- Directivity : $LD_{31} = LI_{31}\text{-}LC = -20\log_{10} |s_{31}|/\beta$
- Directivity : $LD_{24} = LI_{24}\text{-}LC = -20\log_{10} |s_{24}|/\beta$

example : 10 dB waveguide coupler

example : magic T



example : magic T combined with 10dB coupler



Example : E Band 10 dB coupler

Example

LC [dB]	β	α	LA[dB]
40	0.01	0.99995	0.0004
30	0.0316	0.9995	0.004
20	0.1	0.995	0.04
10	0.316	0.948	0.45
3	0.707	0.707	3

Directivity : measure of quality

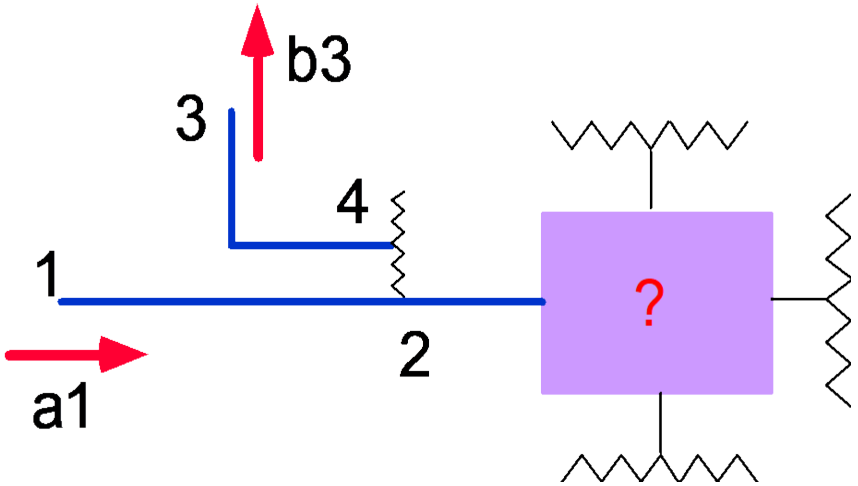
- Microstrip hybrid coupler : 30 dB
- Multihole waveguide couplers : 40 dB
- Waveguide couplers used in metrology : 50-60 dB

Couplers used in measurement

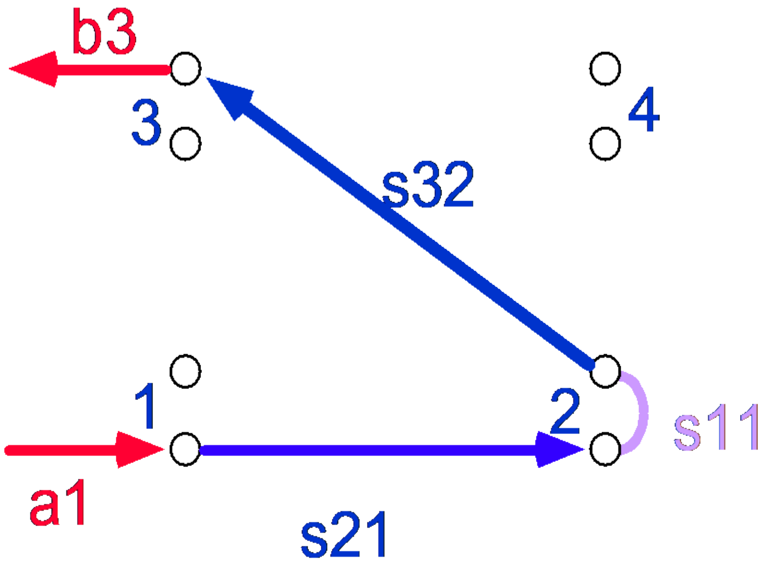
Reflectometry

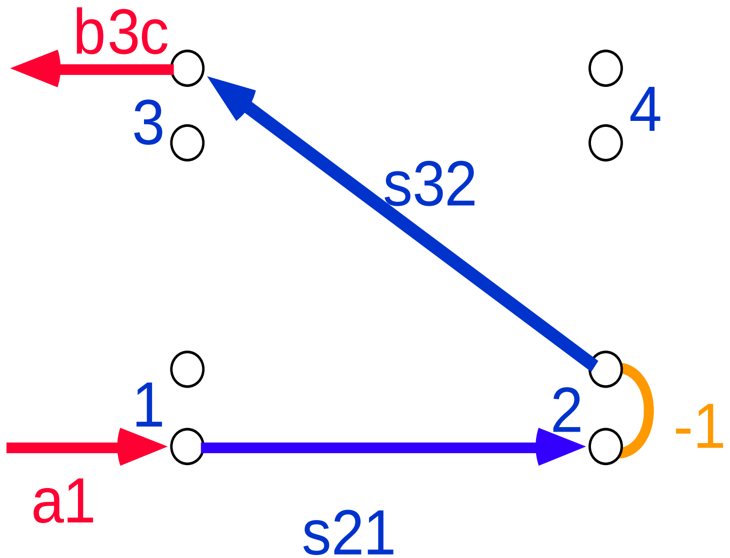
- with 1 coupler
  - amplitude measurements
- with 2 couplers
  - precision measurements
  - phase and amplitude measurements

1 coupler reflectometry



1 coupler reflectometry





$$b_3 = s_{21} s_{ii} s_{32} a_1$$

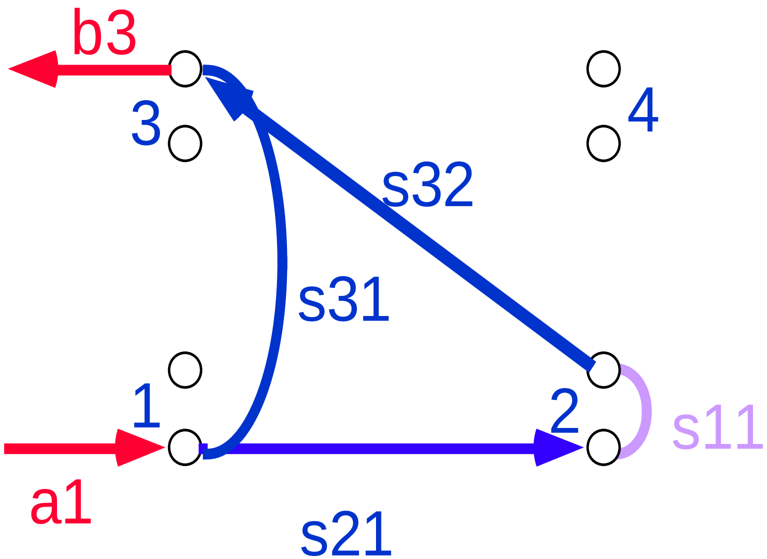
$$\boxed{\frac{b_3}{b_{3c}} = -s_{ii}}$$

$$b_{3c} = -s_{21} s_{32} a_1$$

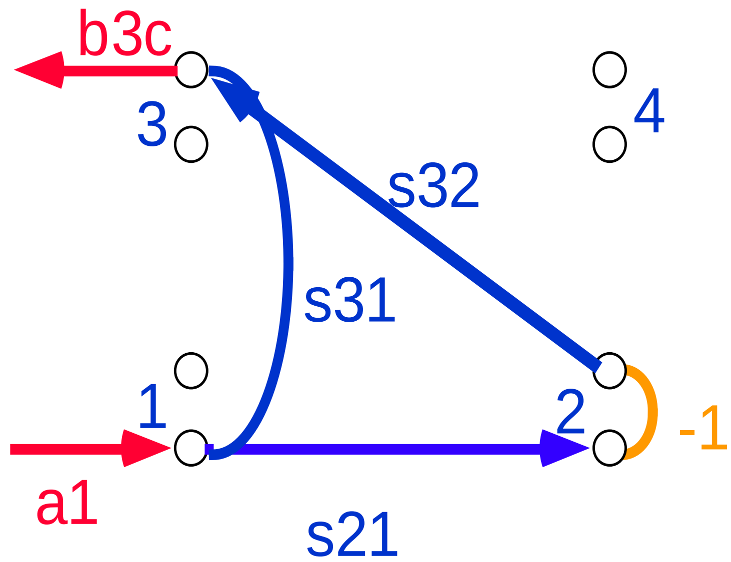
real coupler

- s_{ii}, s_{13}, s_{24} <<1, but non equal to zero
- we need :
  - coupling $LC=-20\log\beta$
  - isolation $LI_{13}=-20\log|s_{13}|$
  - $LI_{14}=-20\log|s_{24}|$
- The quality of the coupler is given by :
  - directivity $LD_{13}=LI_{13}-LC=-20\log|s_{13}|/\beta$
  - $LD_{24}=LI_{24}-LC=-20\log|s_{24}|/\beta$

1 coupler reflectometry



$$b_3 = a_1\left(s_{21}s_{ii}s_{32} + s_{31}\right)$$



$$b_{3c} = a_1\left(-s_{21}s_{32} + s_{31}\right)$$

$$\boxed{\frac{b_3}{b_{3c}} = \frac{s_{21}s_{ii}s_{32} + s_{31}}{-s_{21}s_{32} + s_{31}}}$$

1 coupler reflectometry

- s_{31} is non negligible at the numerator if s_{ii} <<1
- s_{31} is negligible at the denominator if the couple is of good quality : $$\frac{b_3}{b_{3c}} = -s_{ii} - \frac{s_{31}}{s_{21}s_{32}}$$
- The phases of the signals are not known

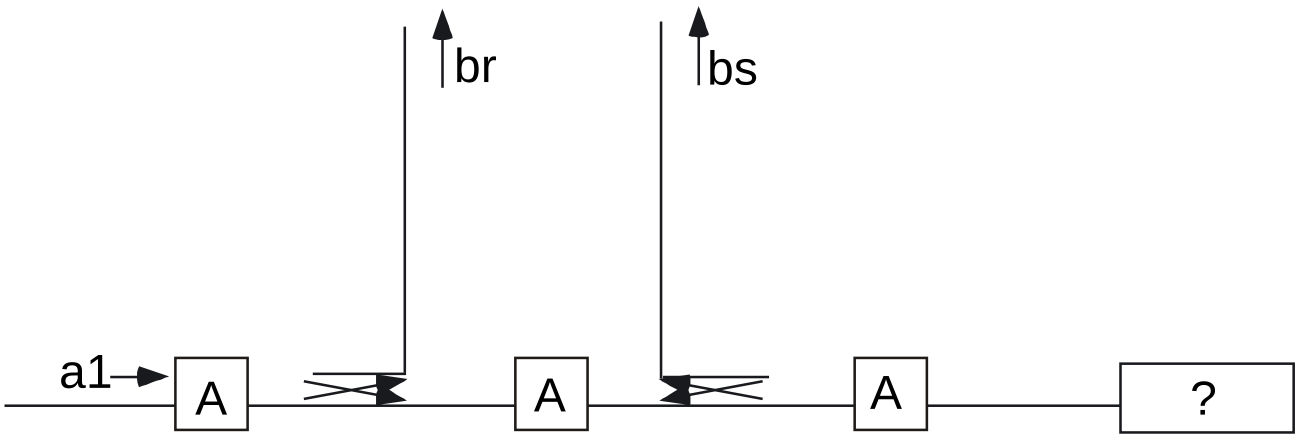
$$\boxed{\left|\frac{b_3}{b_{3c}}\right| - \left|\frac{s_{31}}{s_{21}s_{32}}\right| \le s_{ii} \le \left|\frac{b_3}{b_{3c}}\right| + \left|\frac{s_{31}}{s_{21}s_{32}}\right|}$$

1 coupler reflectometry

- For a weak coupling (LC large) S21 ≈ 1
- The error term is directly the directivity of the coupler : $$\left|\frac{b_3}{b_{3c}}\right| - \left|\frac{s_{31}}{s_{32}}\right| \le s_{ii} \le \left|\frac{b_3}{b_{3c}}\right| + \left|\frac{s_{31}}{s_{32}}\right|$$
-

LD [dB]	$\lvert s_{31}/s_{32} \rvert$
10	0.3162
20	0.1
30	0.03162
40	0.01
50	0.003162
60	0.001

2 coupler reflectometry



$$\frac{b_s}{b_r} = \frac{As_{ii} + B}{Cs_{ii} + D}$$

$$A = s_{21}s_{32} - s_{31}s_{22}$$

$$B = s_{31}$$

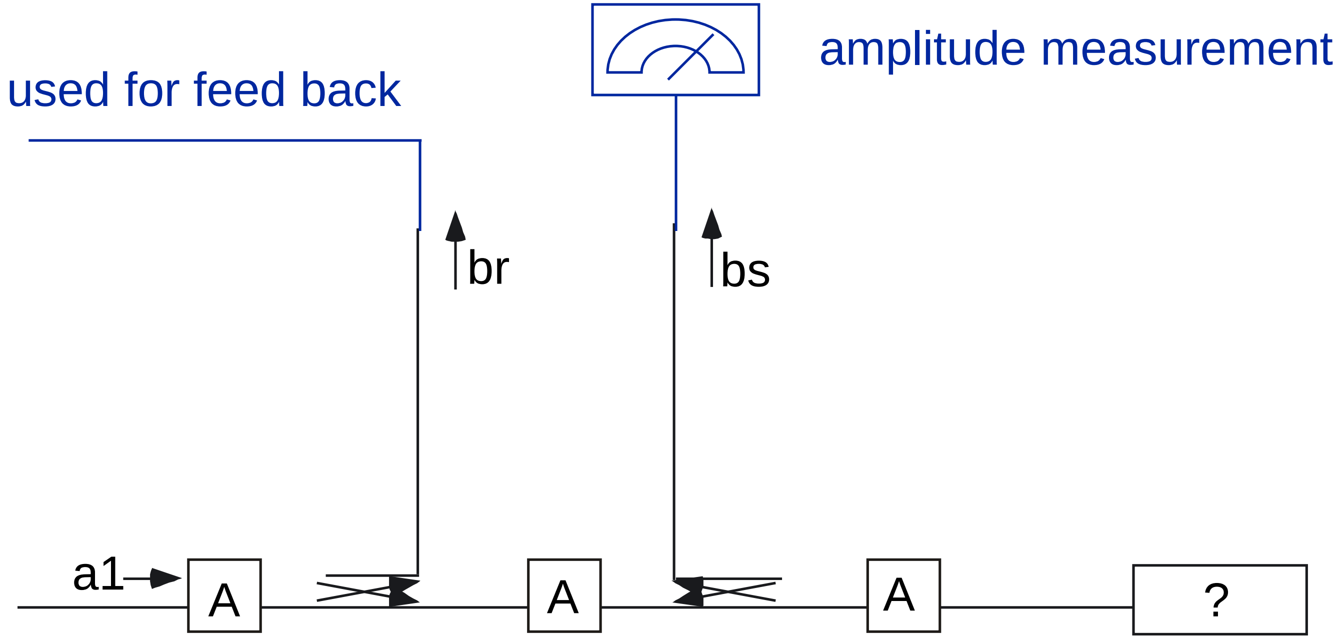
$$C = s_{21}s_{42} - s_{41}s_{22}$$

$$D = s_{41}$$

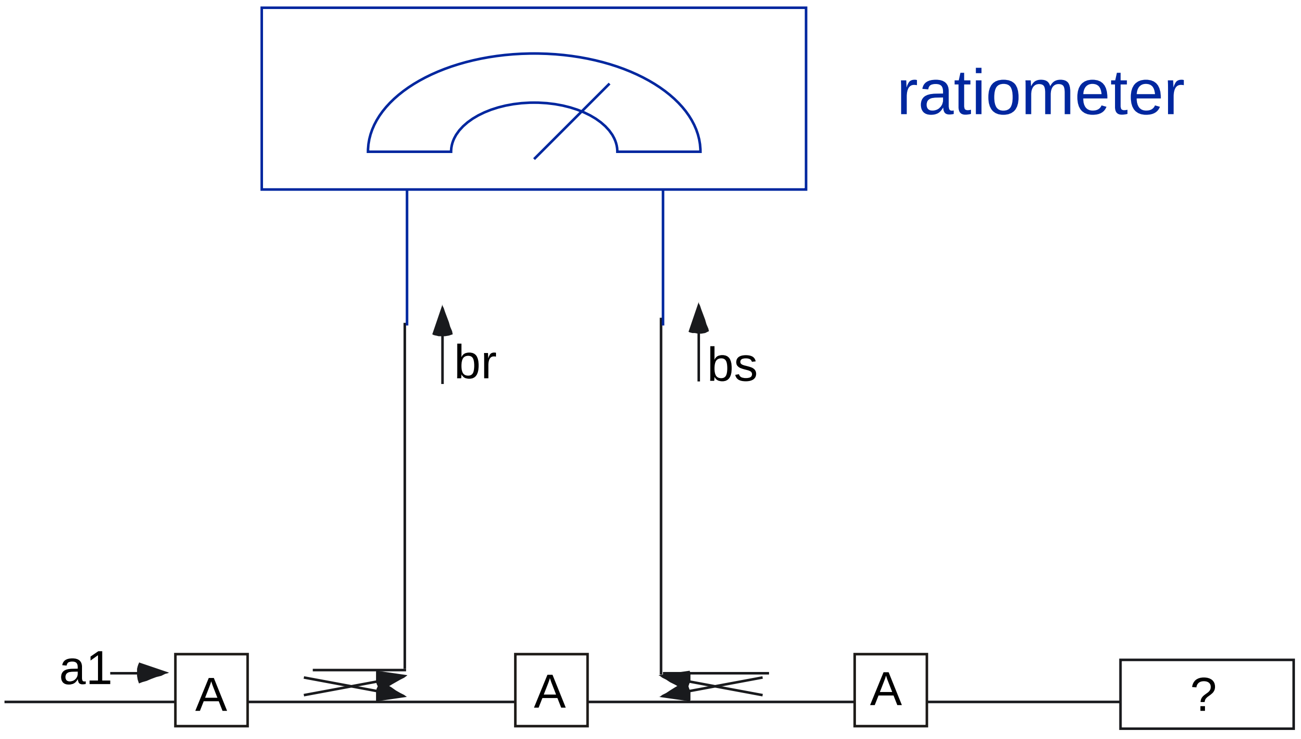
Use adapters to put B=C=0

$$\frac{b_s}{b_r} = \frac{s_{21}s_{ii}s_{32}}{s_{41}}$$

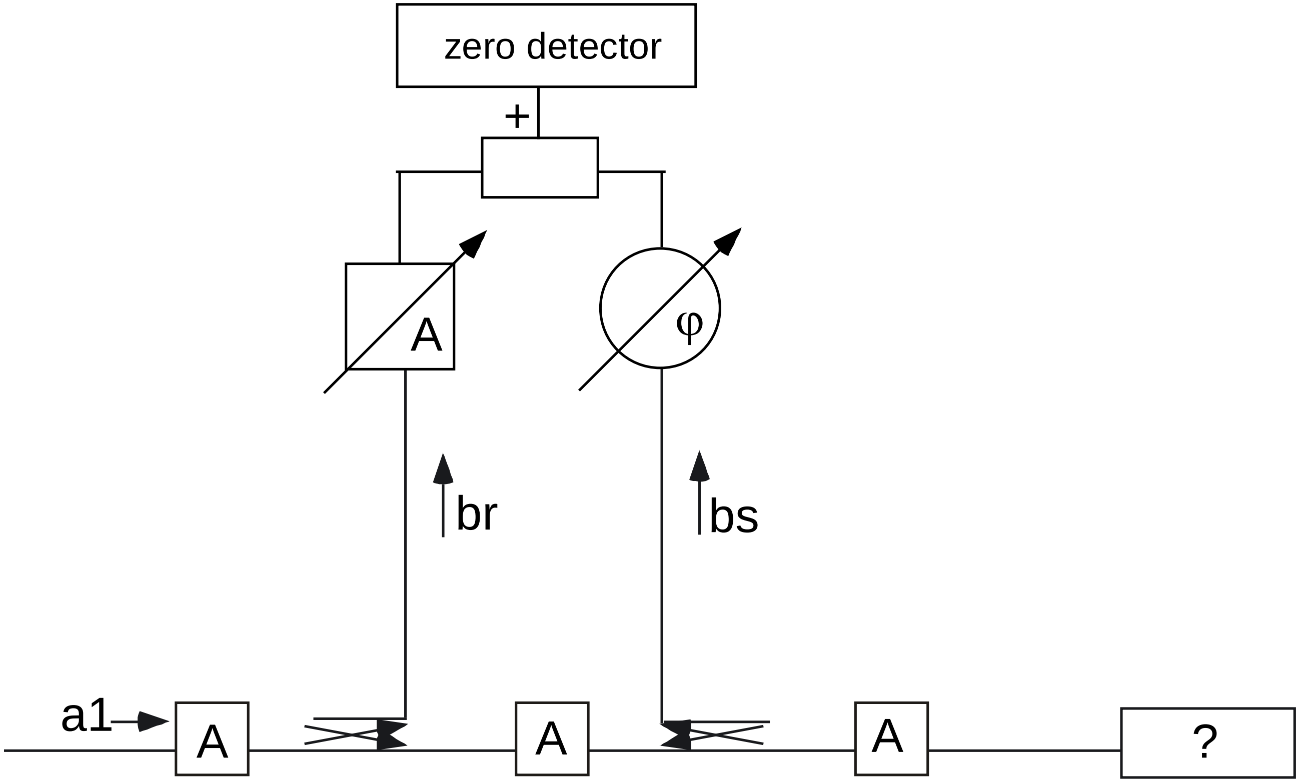
2 coupler reflectometry



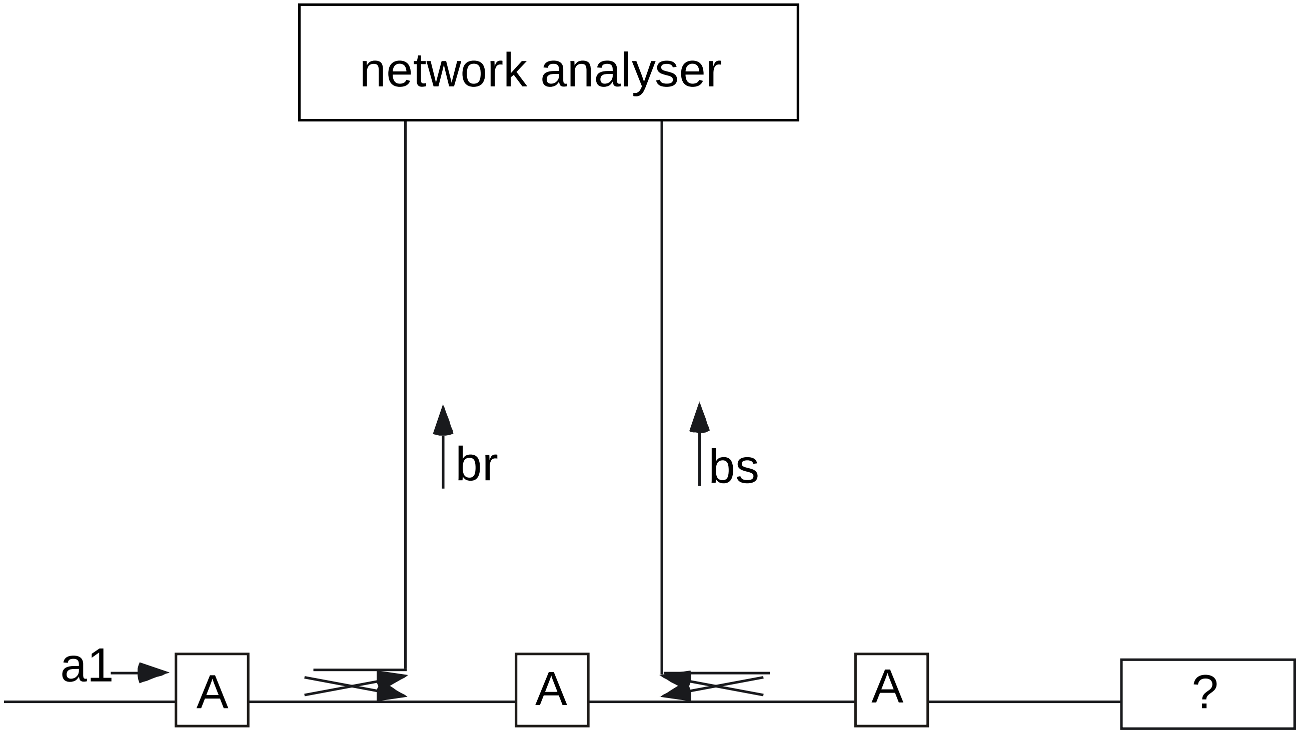
2 coupler reflectometry



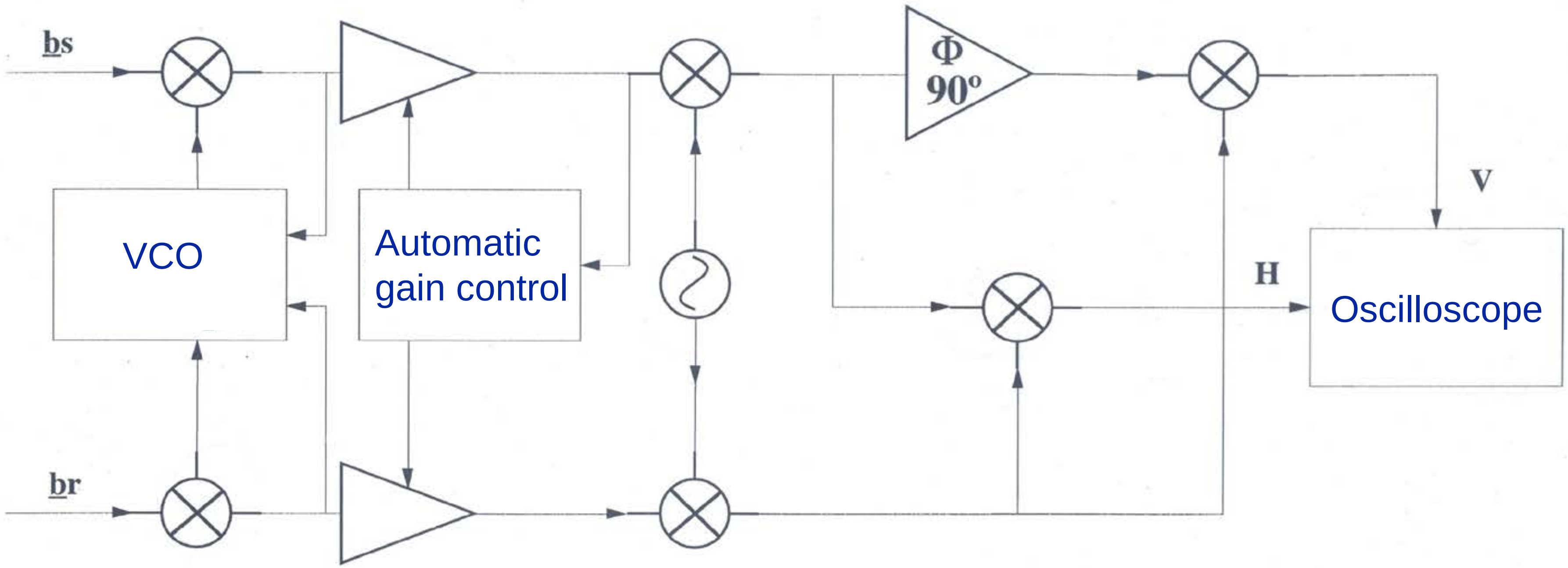
2 coupler reflectometry



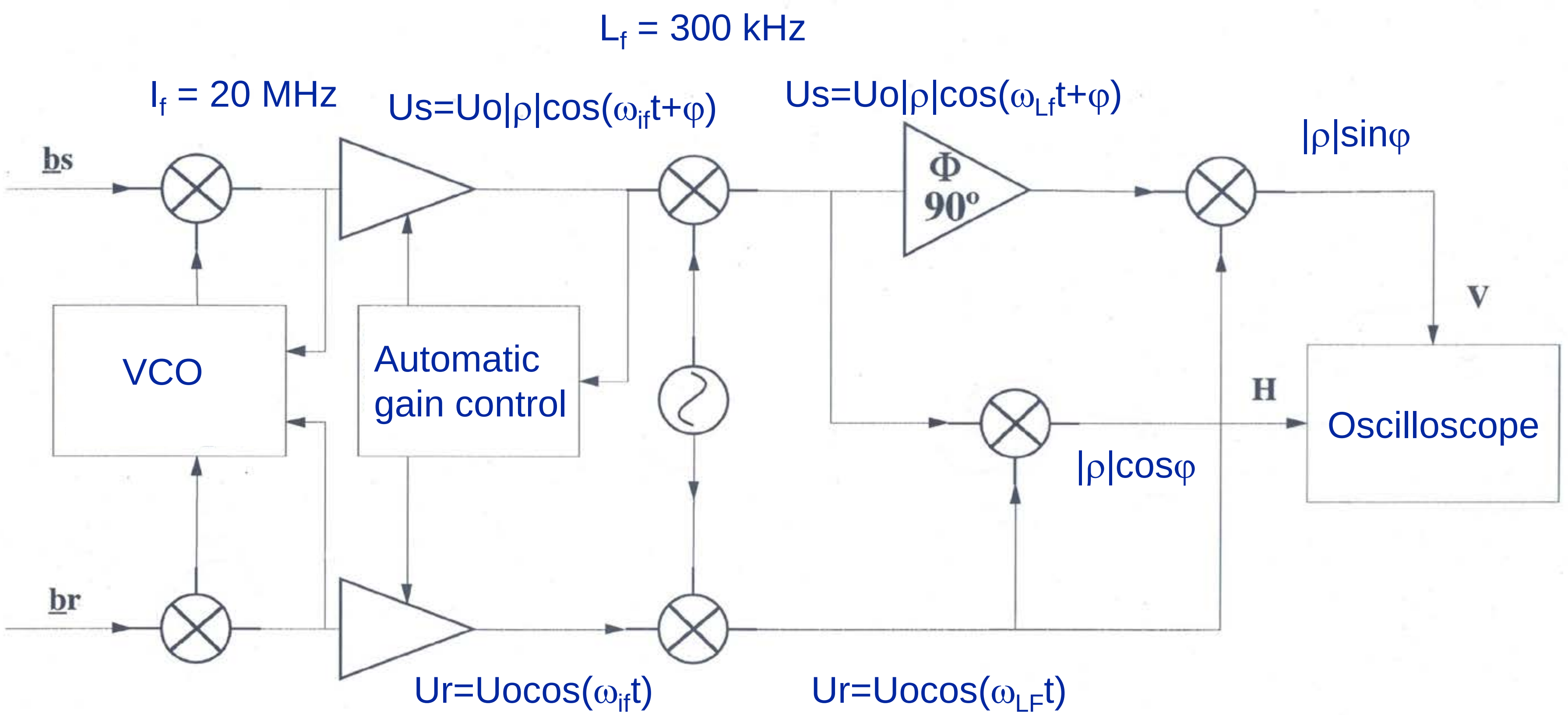
2 coupler reflectometry



Network analyser (principle)



Network analyser (principle)



Transmission measurements

(Attenuation measurements)

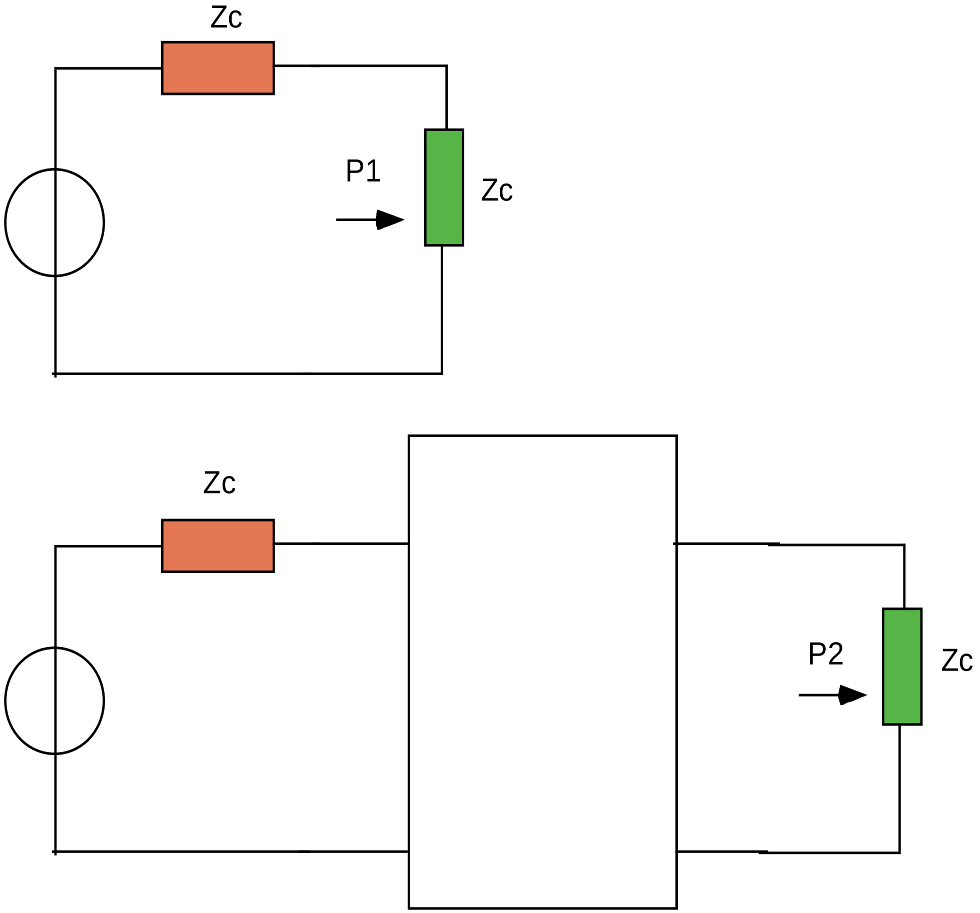
Transmission measurements

- Attenuation (very common)
- Phase (more seldom)

Causes of attenuation

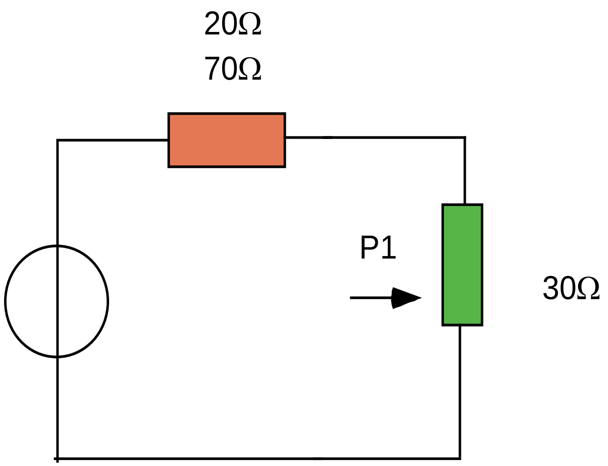
- Absorbtion in the component
- Mismatch of the component

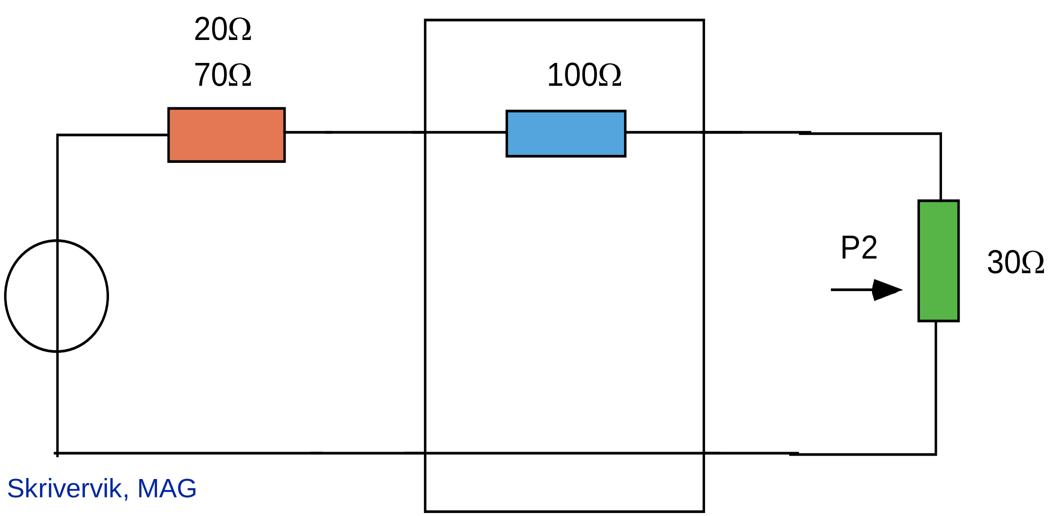
Attenuation



LA=-10 Log(P2/P1)   dB

Attenuation : example





LA=-10 Log(P2/P1) dB

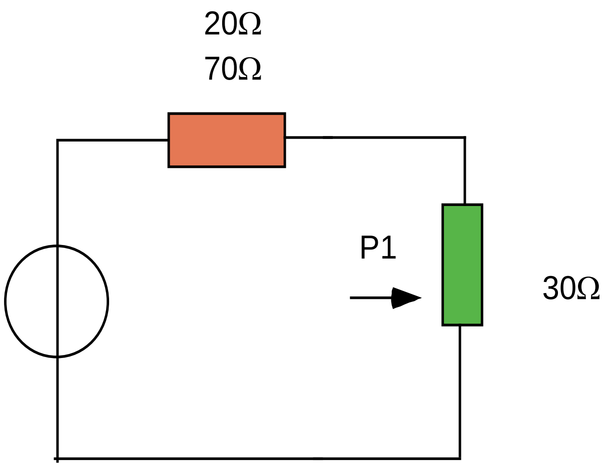
a) Rg = 20 Ω

$$P_1 = \left(\frac{100}{20+30}\right)^2 30 = 120W$$

$$P_2 = \left(\frac{100}{20+30+100}\right)^2 30 = \frac{40}{3}W$$

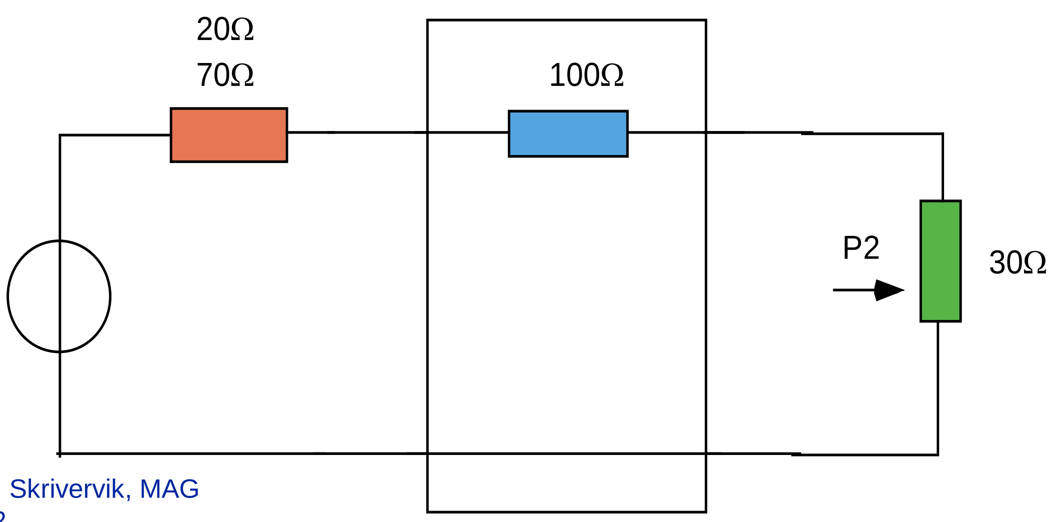
$$LA = 10Log9 = 9.5dB$$

Attenuation : example



LA=-10 Log(P2/P1)   dB

b) Rg = 70 Ω

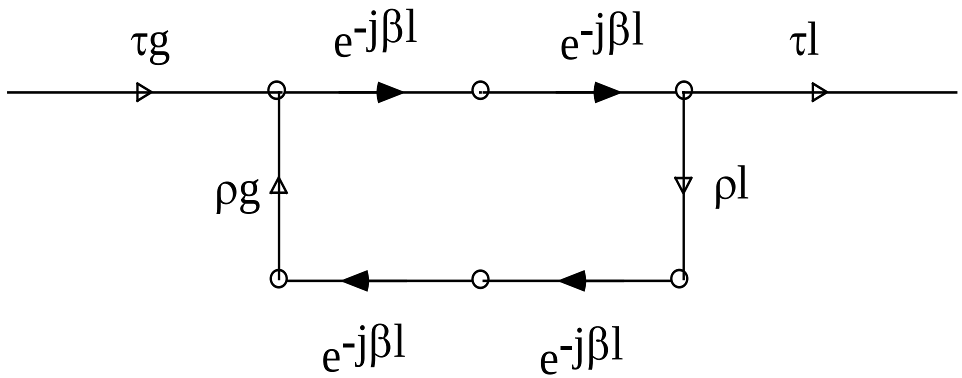


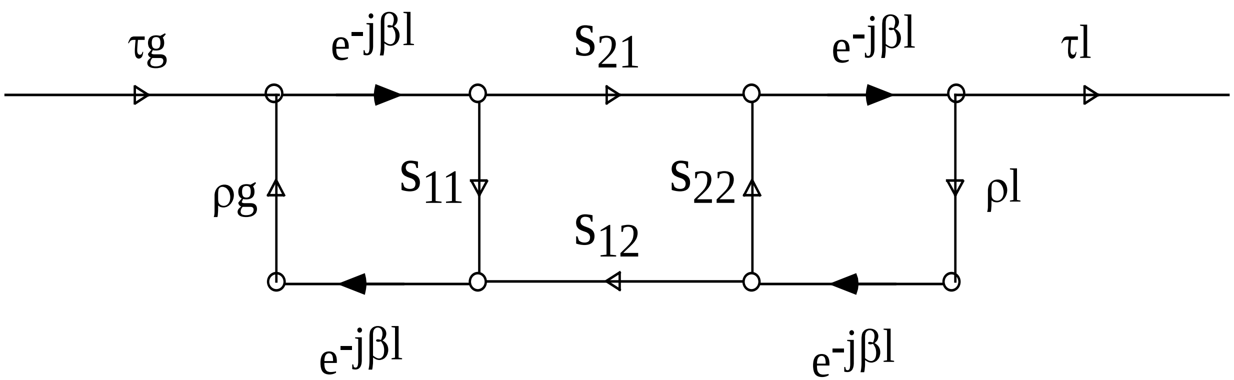
$$P_1 = \left(\frac{100}{70+30}\right)^2 30 = 30W$$

$$P_2 = \left(\frac{100}{70+30+100}\right)^2 30 = \frac{30}{4}W$$

$$LA = 10 Log 4 = 6dB$$

Effet od the mismatch of the source and the load

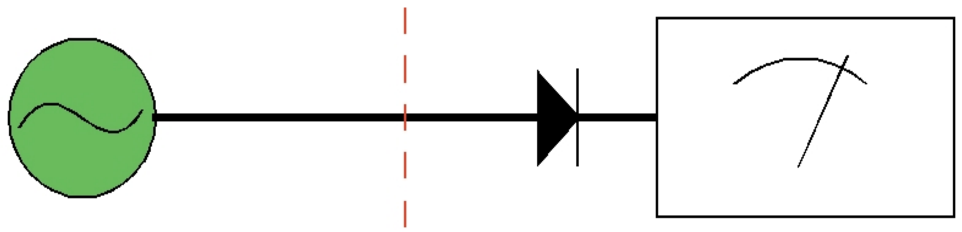


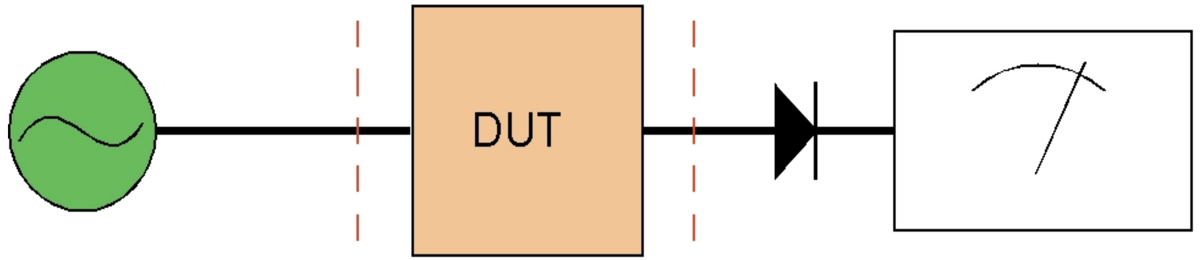


Methods

- Direct measurement
- Substitution measurement
- Measurement through reflection measurement
- Method using couplers

Direct measurement



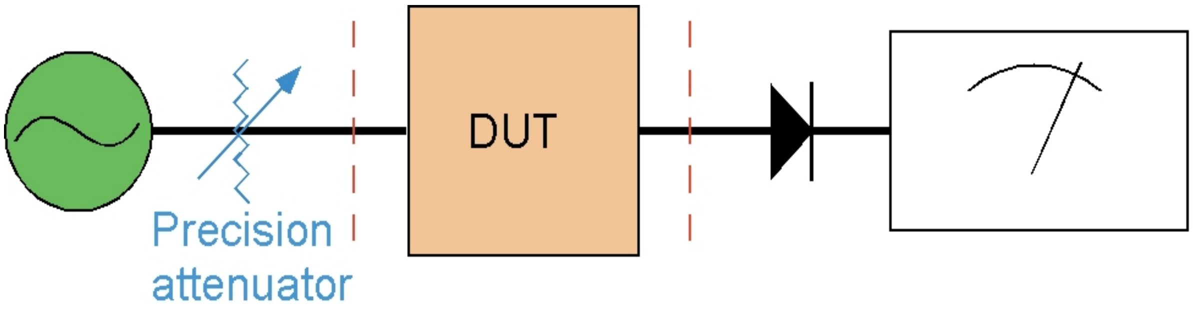


Direct measurement

- The detector should be adapted (use attnenuator or isolator)
- The detector should be quadratic (limited dynamic range)
- The connexions should be reproductible (sometimes difficult)
- The generator should deliver a signal which is stable over time

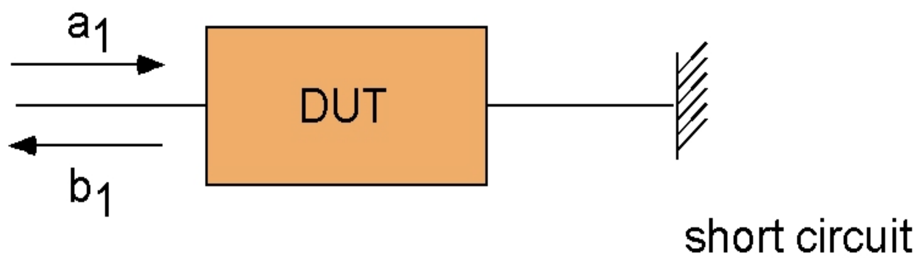
Cheap , but not very precise and quite limited !!!

Substitution measurements

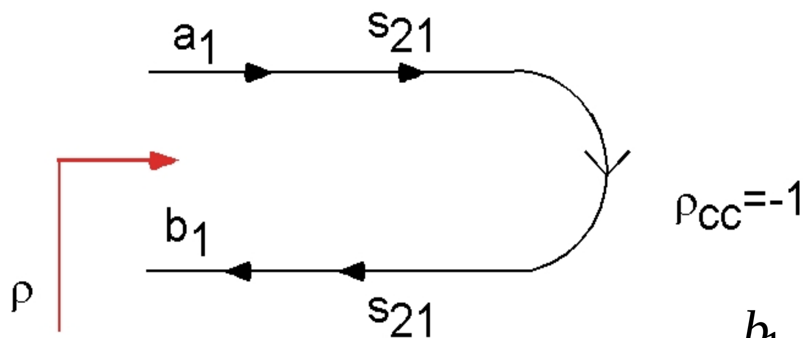


- The precision attenuator (rotative blade) allows a measurement precision od 1-2%, over a 60 dB dynamic range
- Only for wavguides
- No precision attenuators for coaxial cables

Measurement through reflection measurement



1) the device is matched but reciprocal

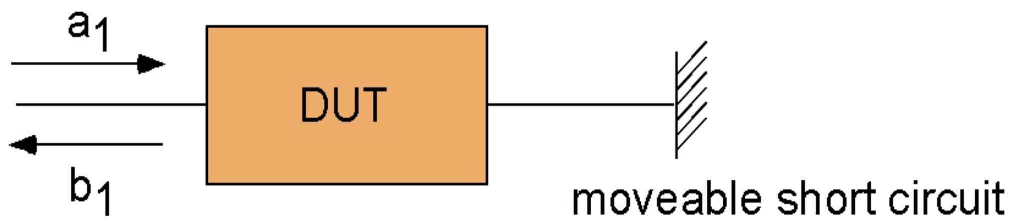


$$\rho = \frac{b_1}{a_1} = s_{21}^2 \rho_{cc} \simeq s_{21}^2$$

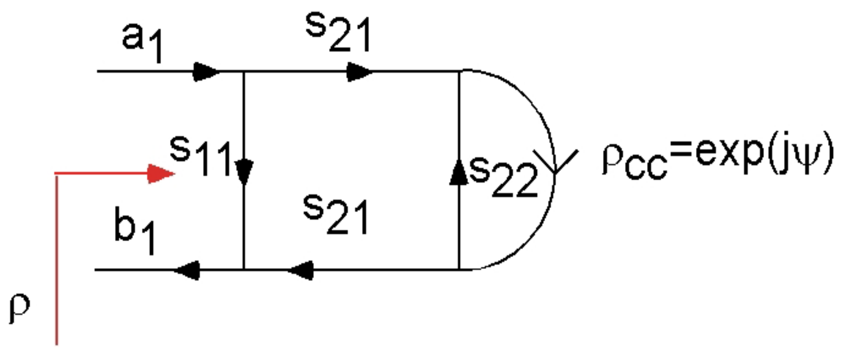
$$LA = -10\log|\rho| + 10\log|\rho_{cc}| \simeq -10\log|\rho|$$

$$\varphi = \frac{1}{2}\arg(\rho) - \frac{1}{2}\arg(\rho_{cc}) \pm n\pi \simeq \frac{1}{2}\arg(\rho) + \frac{\pi}{2} \pm n\pi$$

Measurement through reflection measurement



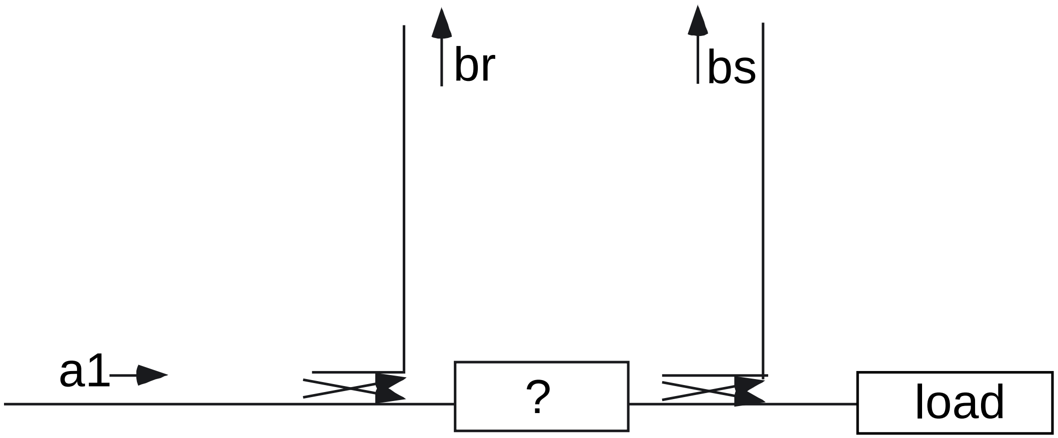
2) the device is mis-matched but reciprocal



$$\rho = \frac{b_1}{a_1} = s_{11} + \frac{s_{21}^2\, e^{-j\psi}}{1 - s_{21}\, e^{-j\psi}}$$

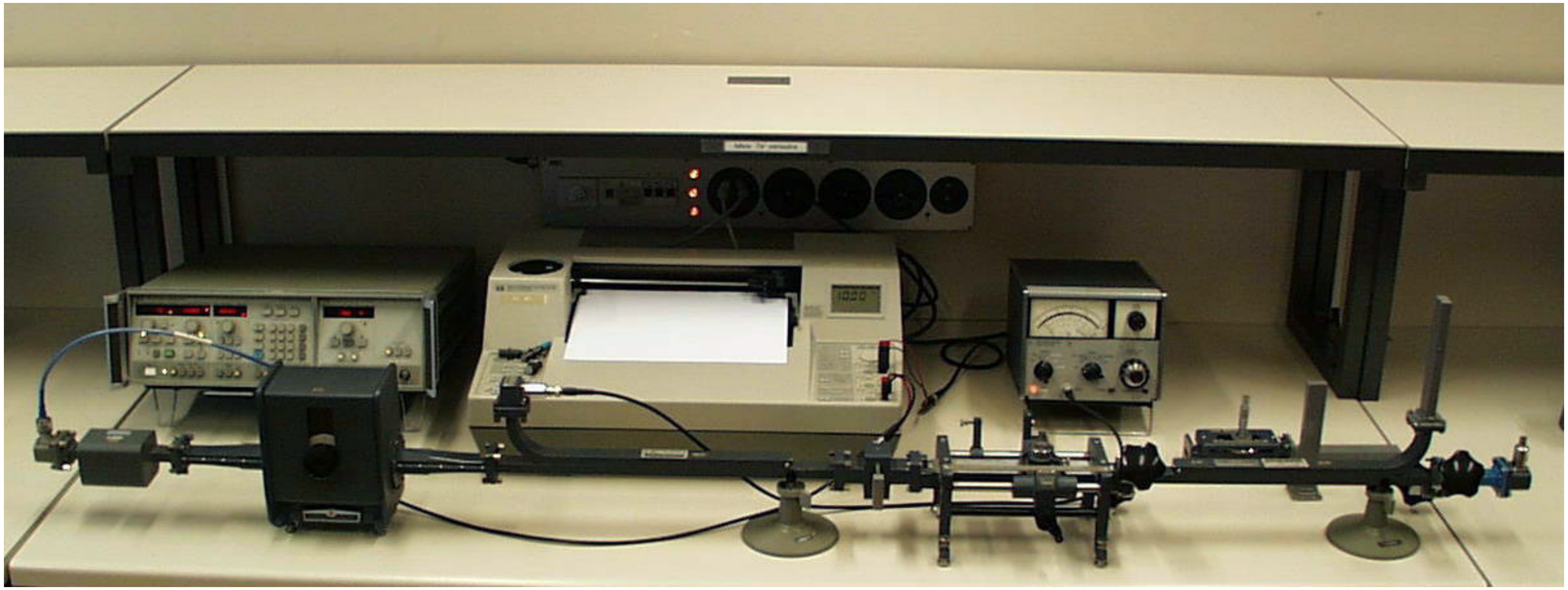
f(ψ) is a circle on the Smith chart

Attenuation measurement



Same principle than reflectometry

Microwave measurements



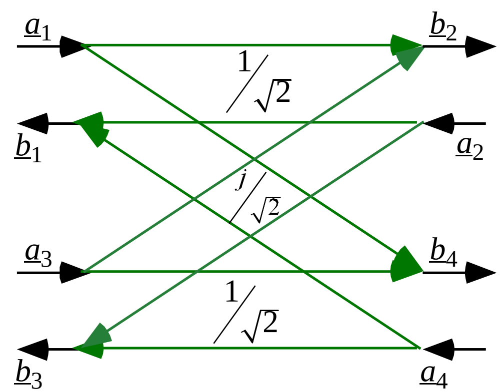
Network Analysor

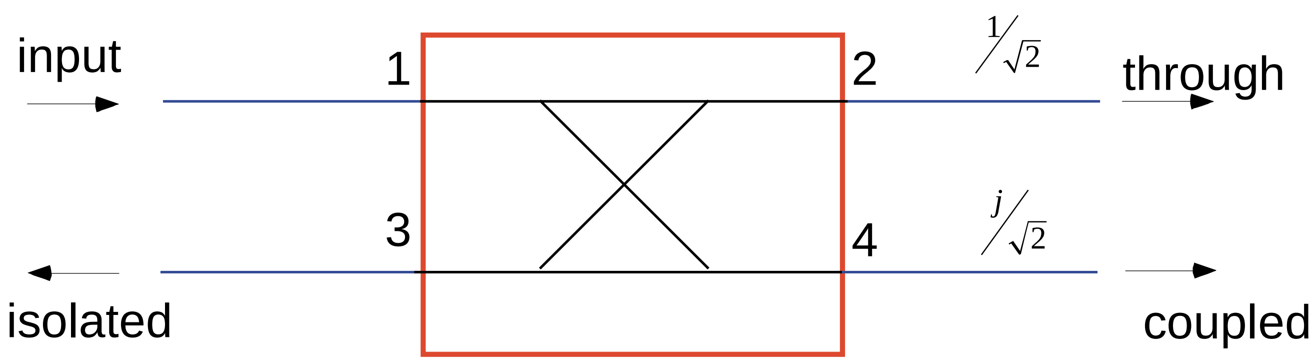
Examples of couplers in printed technology

- the quadrature hybrid
- the 180° hybrid
- examples of utilization

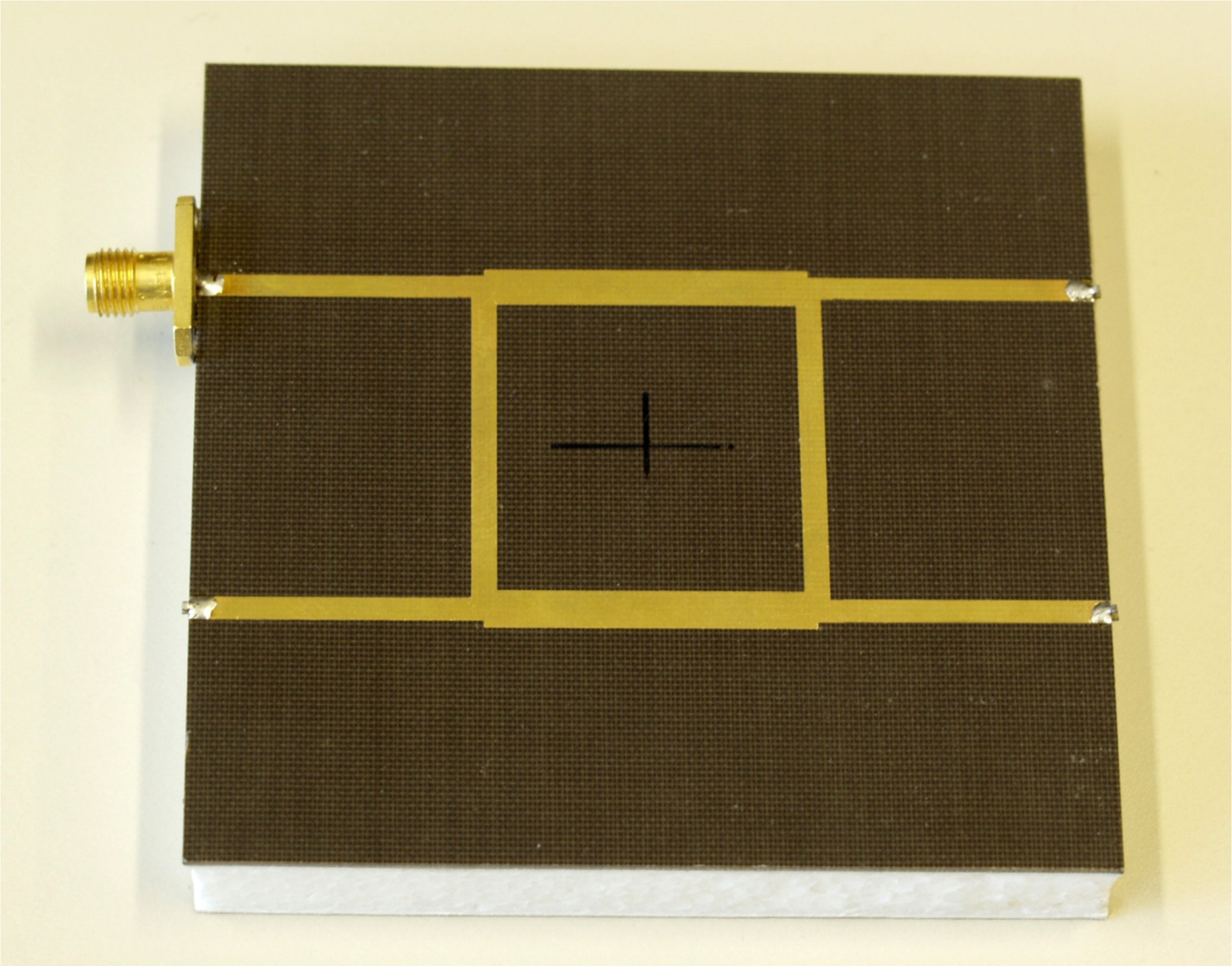
The quadrature hybrid

$$[S]=\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 & j \\ 1 & 0 & j & 0 \\ 0 & j & 0 & 1 \\ j & 0 & 1 & 0 \end{bmatrix}$$

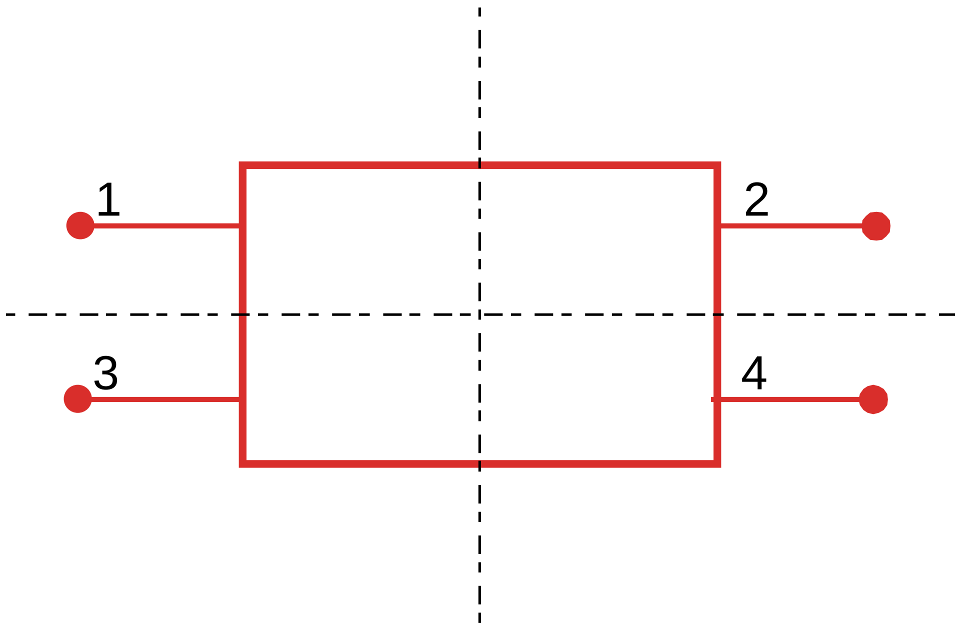




The quadrature hybrid



quadriports with double symmetry (1)



$$\begin{bmatrix} S_1 & S_2 & S_3 & S_4 \\ S_2 & S_1 & S_4 & S_3 \\ S_3 & S_4 & S_1 & S_2 \\ S_4 & S_3 & S_2 & S_1 \end{bmatrix}$$

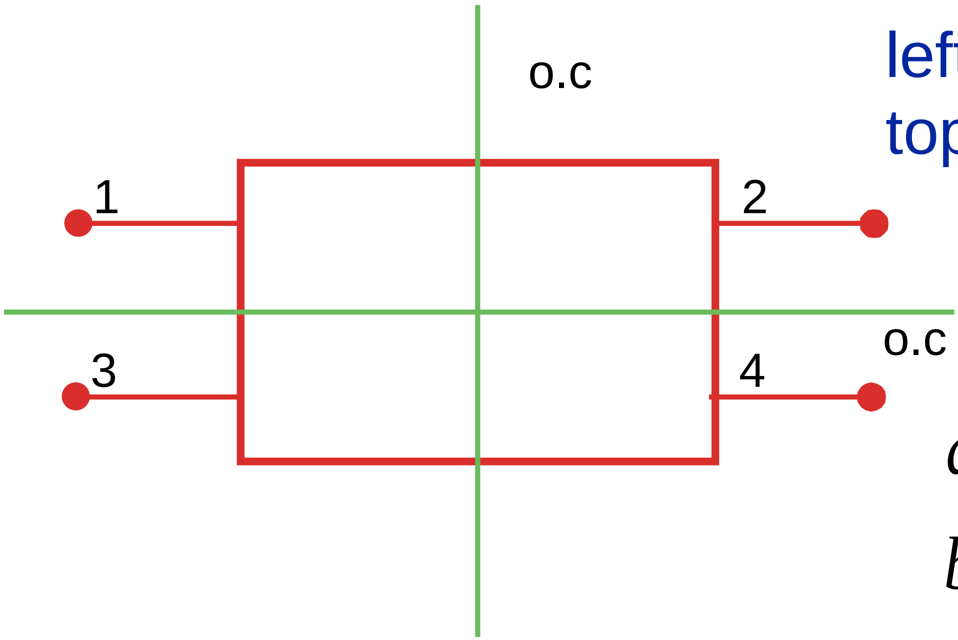
$$S_1 = S_{11} = S_{22} = S_{33} = S_{44}$$

$$S_2 = S_{12} = S_{21} = S_{34} = S_{43}$$

$$S_3 = S_{13} = S_{31} = S_{42} = S_{24}$$

$$S_4 = S_{14} = S_{41} = S_{32} = S_{23}$$

quadriports with double symmetry (2)



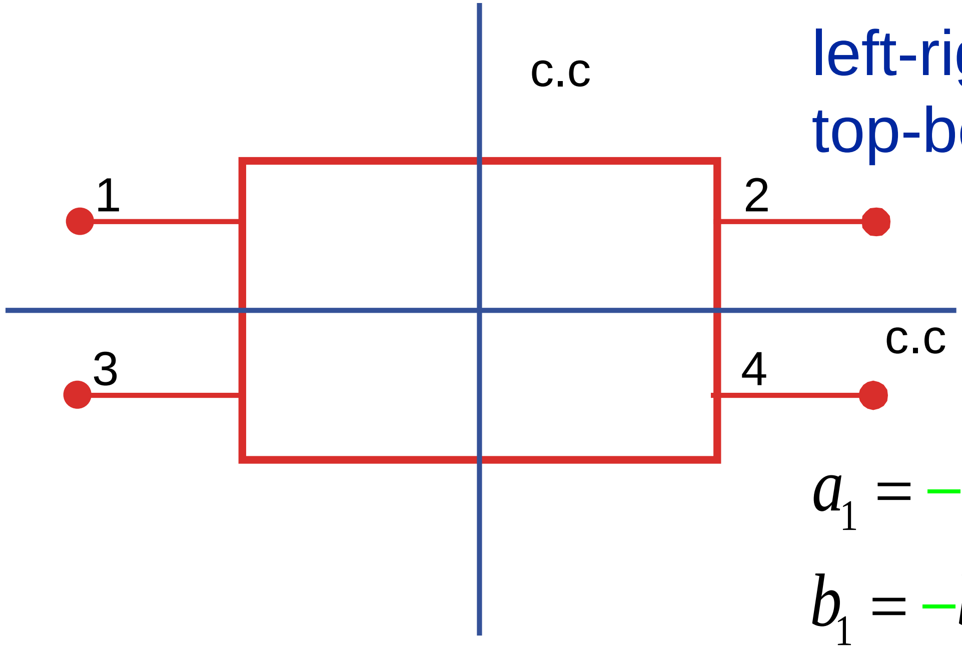
left-right : symmetric excitation
top-bottom : symmetric excitation

$$a_1 = a_2 = a_3 = a_4$$

$$b_1 = b_2 = b_3 = b_4$$

$$\rho_{ss} = \frac{b_{ss}}{a_{ss}} = s_1 + s_2 + s_3 + s_4$$

quadriports with double symmetry (3)



left-right : antisymmetric excitation

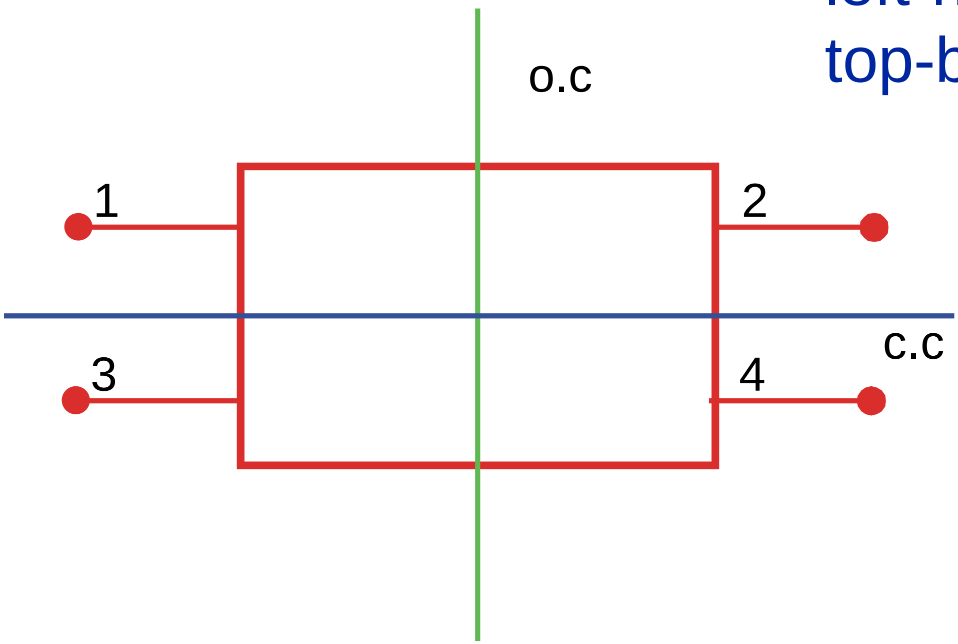
top-bottom : antisymmetric excitation

$$a_1 = -a_2 = -a_3 = a_4$$

$$b_1 = -b_2 = -b_3 = b_4$$

$$\rho_{aa} = \frac{b_{aa}}{a_{aa}} = s_1 - s_2 - s_3 + s_4$$

quadriports with double symmetry (4)



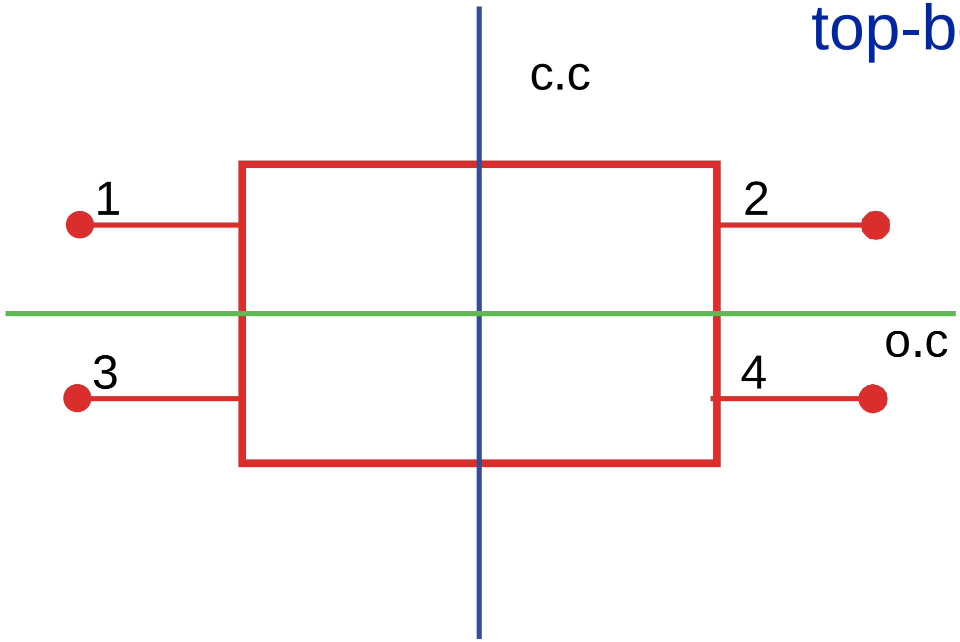
left-right : symmetric excitation
top-bottom : antisymmetric excitation

$$a_1 = a_2 = -a_3 = -a_4$$

$$b_1 = b_2 = -b_3 = -b_4$$

$$\rho_{sa} = \frac{b_{sa}}{a_{sa}} = s_1 + s_2 - s_3 - s_4$$

quadriports with double symmetry (5)



left-right : antisymmetric excitation
top-bottom excitation

$$a_1 = -a_2 = a_3 = -a_4$$

$$b_1 = -b_2 = b_3 = -b_4$$

$$\rho_{as} = \frac{b_{as}}{a_{as}} = s_1 - s_2 + s_3 - s_4$$

quadriports with double symmetry (6)

$$\begin{bmatrix} \rho_{ss} \\ \rho_{as} \\ \rho_{sa} \\ \rho_{aa} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} \rho_{ss} \\ \rho_{as} \\ \rho_{sa} \\ \rho_{aa} \end{bmatrix}$$

quadriports with double symmetry (7)

matching : s_1=0 $$\rho_{ss}+\rho_{as}+\rho_{sa}+\rho_{aa}=0$$

directivity : s_3=0 $$\rho_{ss}+\rho_{as}-\rho_{sa}-\rho_{aa}=0$$

$$\rho_{as}=-\rho_{ss} \Rightarrow \arg(\rho_{as})-\arg(\rho_{ss})=\pi$$
$$\rho_{aa}=-\rho_{sa} \Rightarrow \arg(\rho_{aa})-\arg(\rho_{sa})=\pi$$

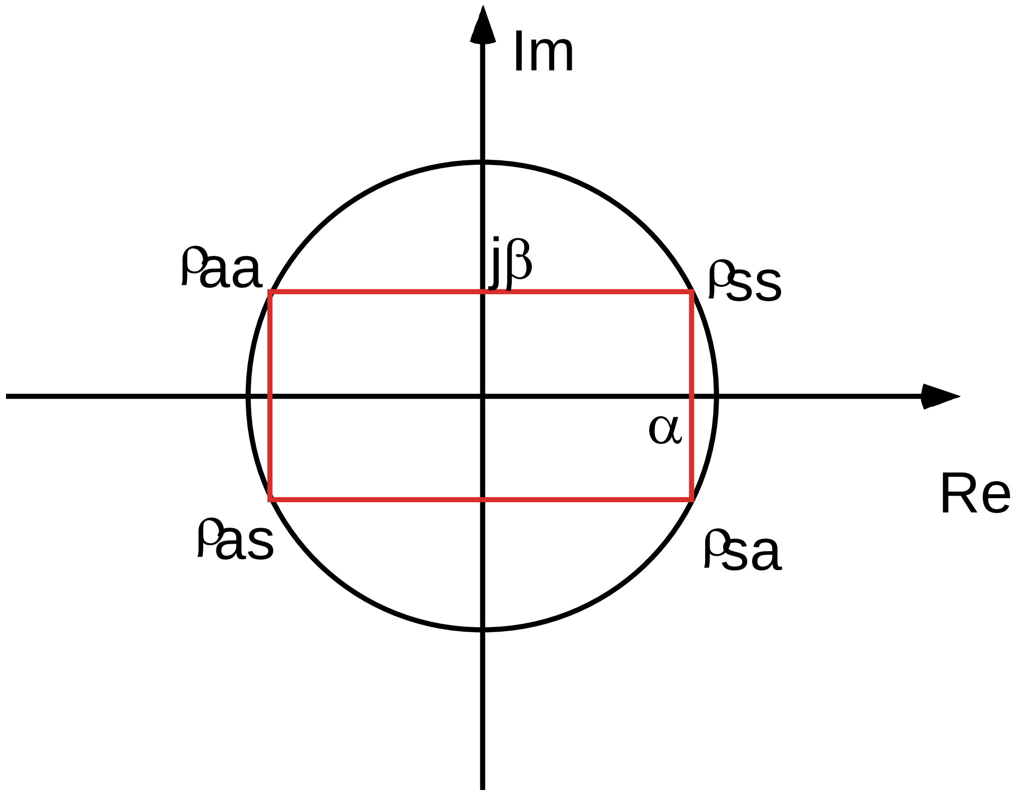
quadriports with double symmetry (7)

Lossless case : $\left|\rho_{ij}\right| = 1$

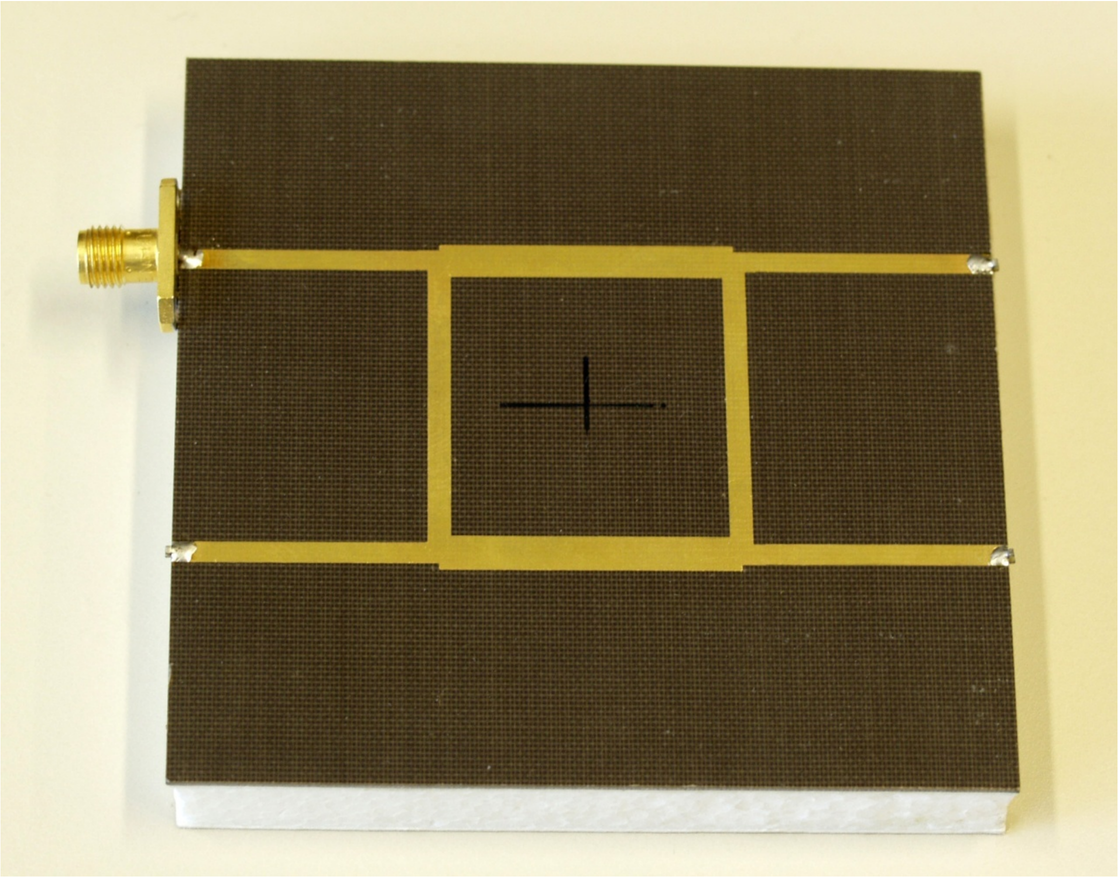
by choice of reference planes

$$s_2 = \frac{\left(\rho_{ss} - \rho_{aa}\right)}{2} = \alpha$$

$$s_4 = \frac{\left(\rho_{ss} + \rho_{aa}\right)}{2} = \beta$$



application to printed hybrid



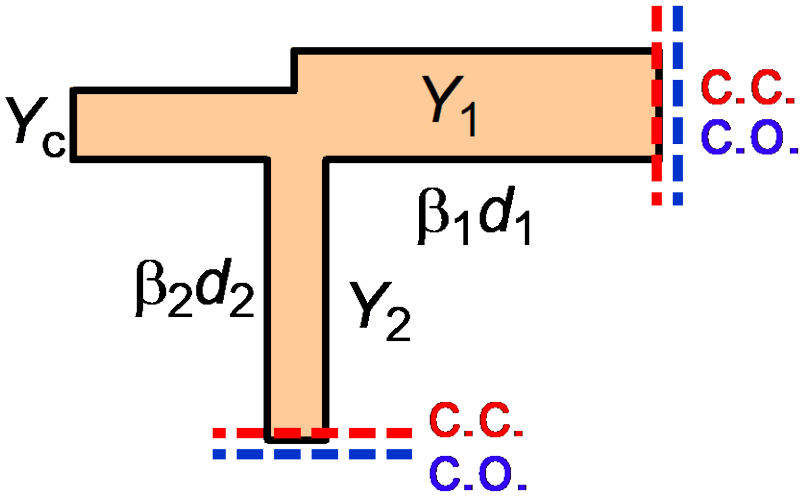
The quadrature hybrid

$$\rho_{ss} = \frac{Y_c - jY_1\tan(\beta_1 d_1) - jY_2\tan(\beta_2 d_2)}{Y_c + jY_1\tan(\beta_1 d_1) + jY_2\tan(\beta_2 d_2)} e^{j\varphi}$$

$$\rho_{as} = \frac{Y_c + jY_1\cot(\beta_1 d_1) - jY_2\tan(\beta_2 d_2)}{Y_c - jY_1\cot(\beta_1 d_1) + jY_2\tan(\beta_2 d_2)} e^{j\varphi}$$

$$\rho_{sa} = \frac{Y_c - jY_1\tan(\beta_1 d_1) + jY_2\cot(\beta_2 d_2)}{Y_c + jY_1\tan(\beta_1 d_1) - jY_2\cot(\beta_2 d_2)} e^{j\varphi}$$

$$\rho_{aa} = \frac{Y_c + jY_1\cot(\beta_1 d_1) + jY_2\cot(\beta_2 d_2)}{Y_c - jY_1\cot(\beta_1 d_1) - jY_2\cot(\beta_2 d_2)} e^{j\varphi}$$



with $\rho_{ss} = -\rho_{as} = \alpha + j\beta$

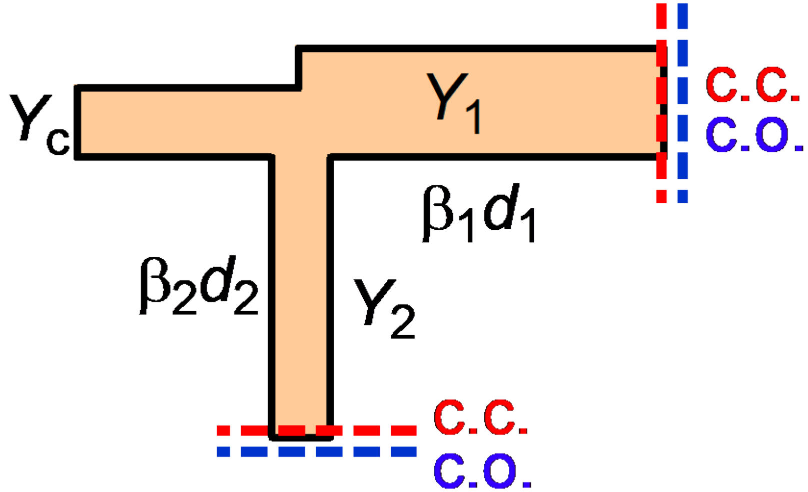
$\rho_{aa} = -\rho_{sa} = -\alpha + j\beta$

The quadrature hybrid

To obtain S1=0, we choose

$$\varphi = \pi/2$$

$$Y_1^2 - Y_2^2 = Y_c^2$$



we get :

$$Y_1 \cot(2\beta_1 d_1) + Y_2 \cot(2\beta_2 d_2) = 0$$

The quadrature hybrid

"easy" solution

$$\beta_1 d_1 = \beta_2 d_2 = \pi/4 \qquad Y_1 = Y_c/\alpha \qquad Y_2 = -jY_c\beta/\alpha$$

β takes a negative value. Lenghts : $d_1 = \lambda_1/8 \quad d_2 = \lambda_2/8$

Hybrid coupler (3 dB): $\alpha = 1/\sqrt{2}$ $\quad Y_1 = \sqrt{2}\,Y_c \quad Y_2 = Y_c$

$\beta = j/\sqrt{2}$

The quadrature hybrid

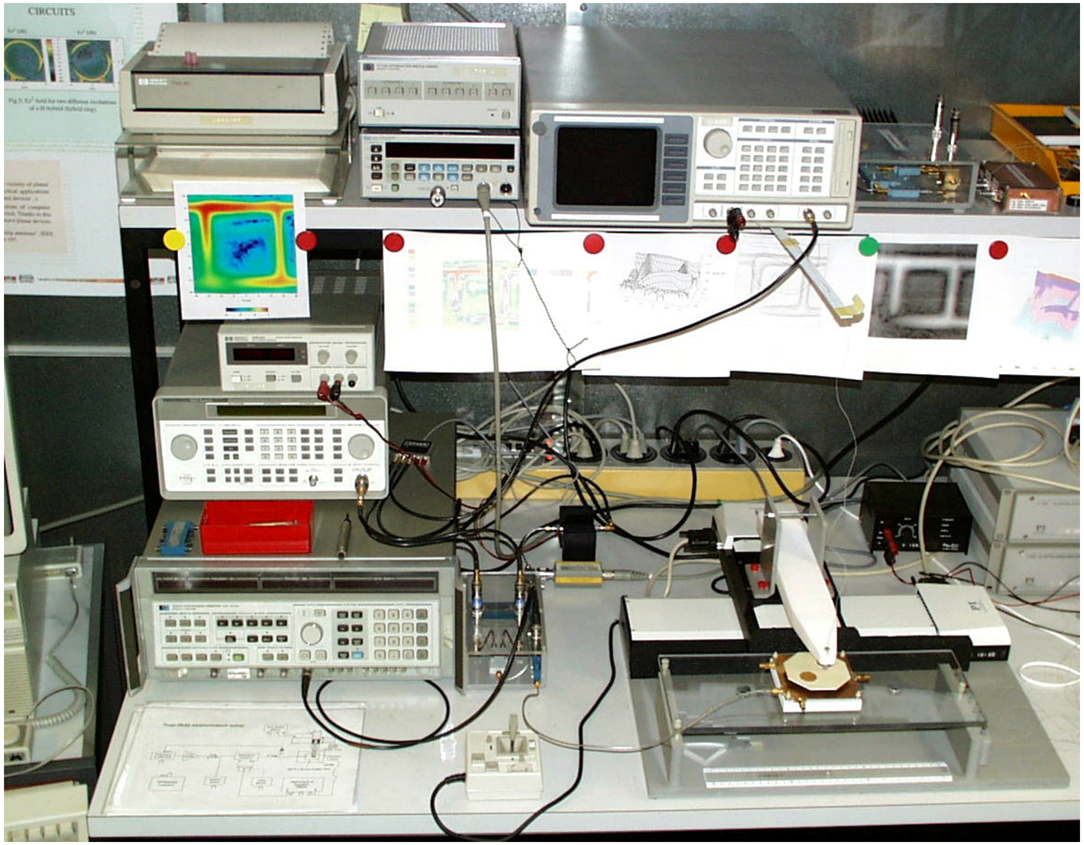
"not so easy" solution

$$Y_1 = \frac{Y_c}{\sqrt{1 - \left[\cot(2\beta_1 d_1) \big/ \cot(2\beta_2 d_2)\right]^2}} \qquad Y_2 = \frac{Y_c}{\sqrt{\left[\cot(2\beta_2 d_2) \big/ \cot(2\beta_1 d_1)\right]^2 - 1}}$$

$$\alpha = \frac{2Y_c\left[Y_1 \tan(\beta_1 d_1) + Y_2 \tan(\beta_2 d_2)\right]}{Y_c^2 + \left[Y_1 \tan(\beta_1 d_1) + Y_2 \tan(\beta_2 d_2)\right]^2}$$

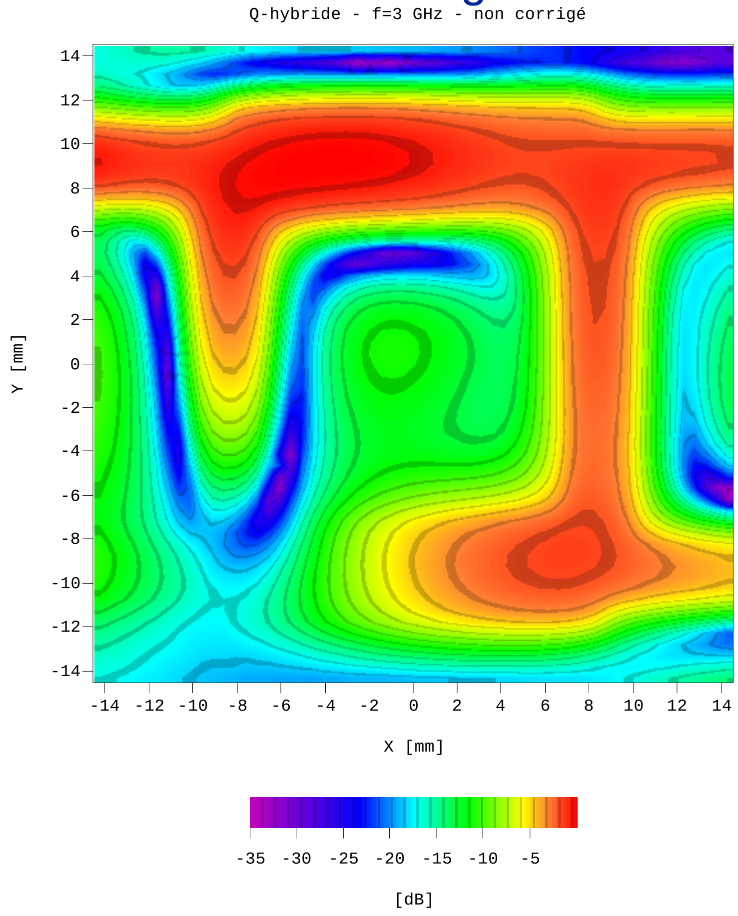
$$\beta = \frac{Y_c^2 - \left[Y_1 \tan(\beta_1 d_1) + Y_2 \tan(\beta_2 d_2)\right]^2}{Y_c^2 + \left[Y_1 \tan(\beta_1 d_1) + Y_2 \tan(\beta_2 d_2)\right]^2}$$

measured verification

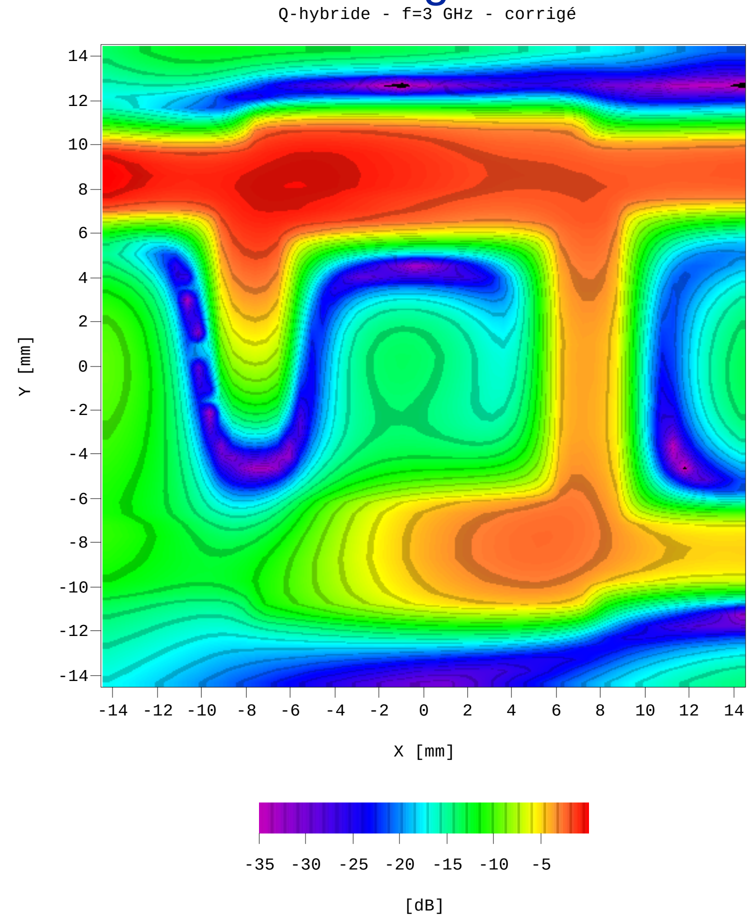


Near field measurements at 3 GHz

non corrigé

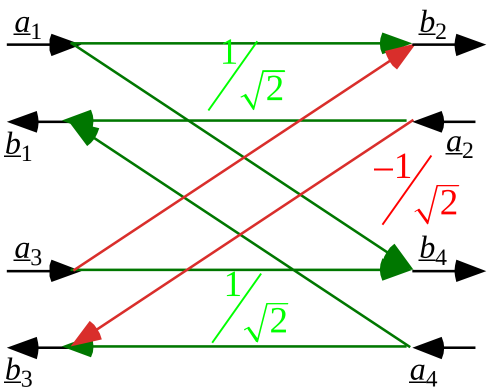


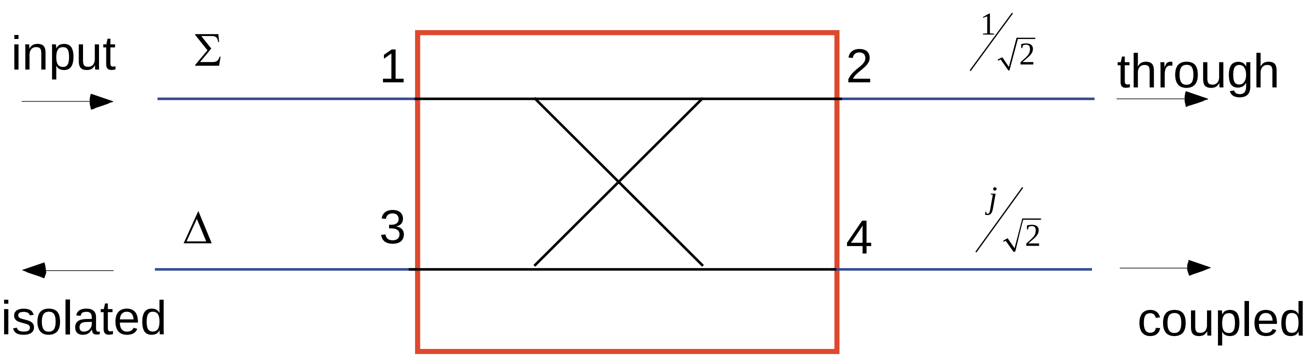
corrigé



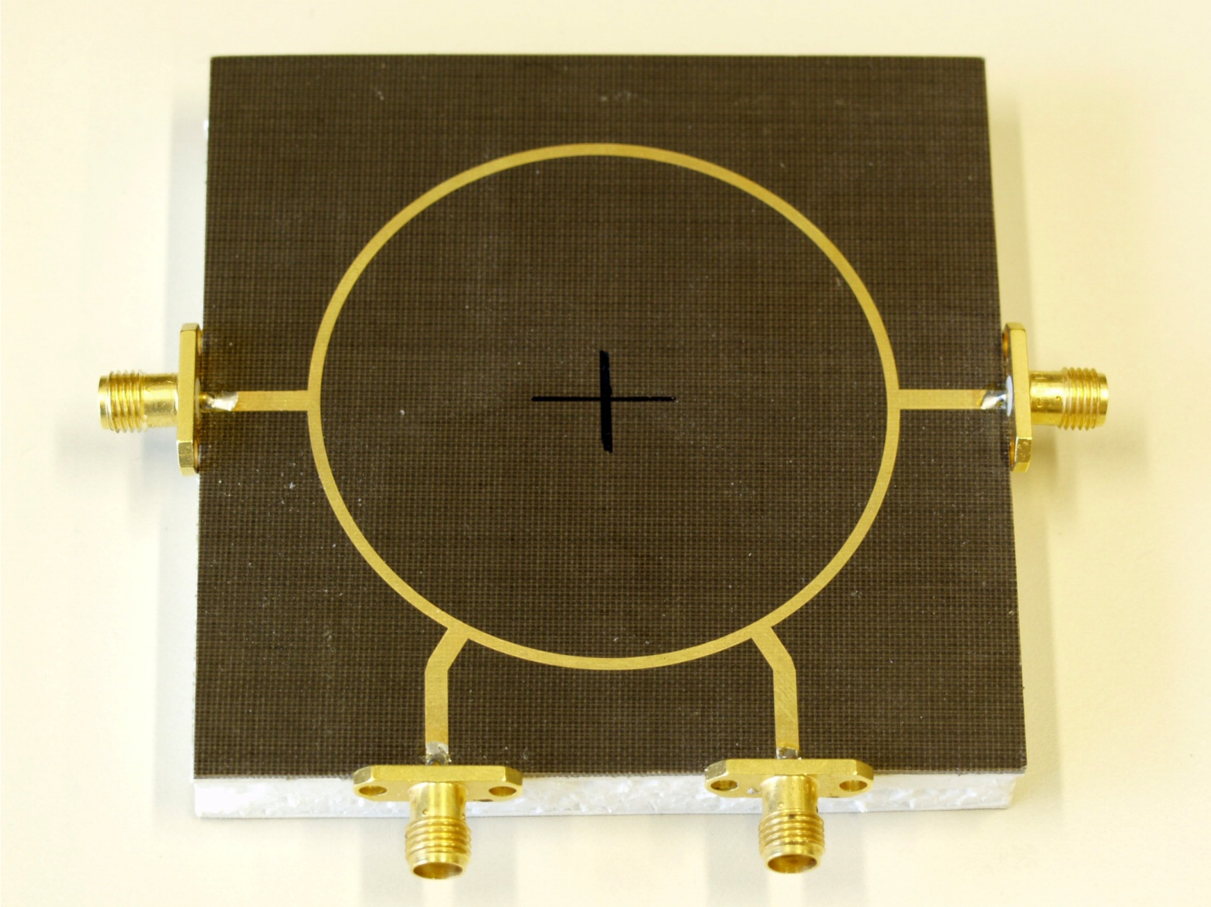
The 180° Hybrid

$$[S]=\frac{1}{\sqrt{2}}\begin{bmatrix}0 & 1 & 0 & 1\\ 1 & 0 & -1 & 0\\ 0 & -1 & 0 & 1\\ 1 & 0 & 1 & 0\end{bmatrix}$$

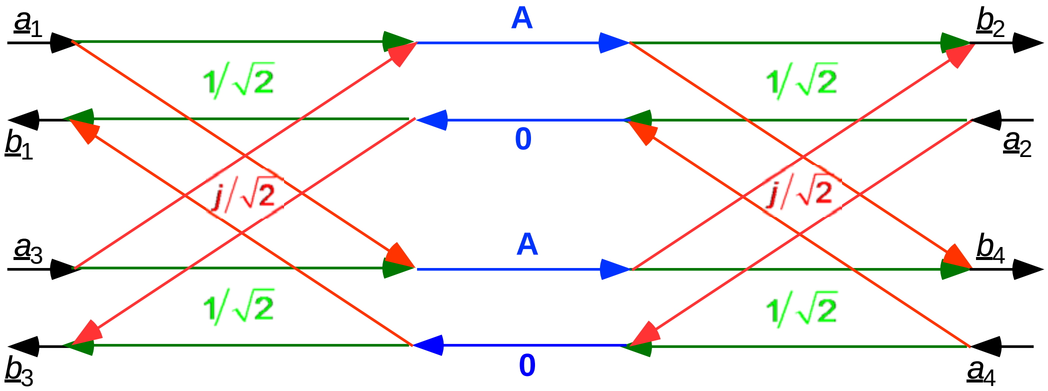




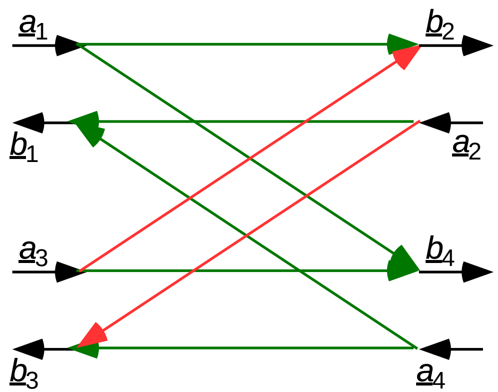
The printed 180° Hybrid



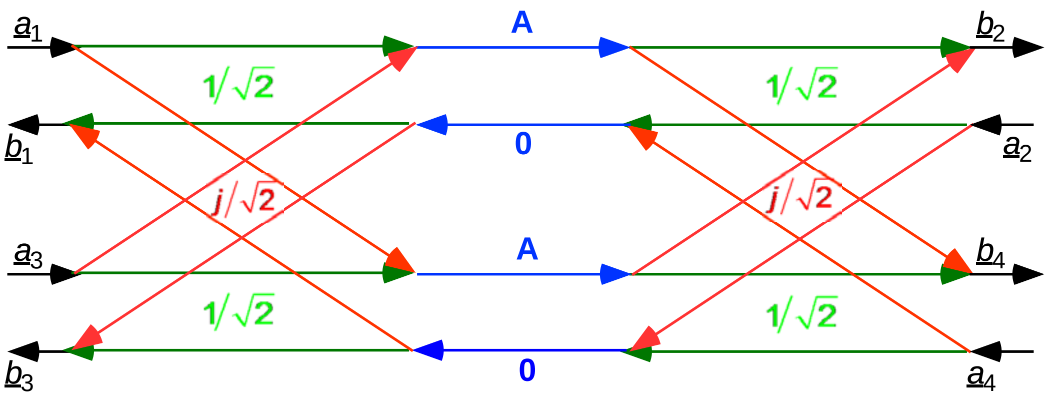
rat race
Hybrid ring



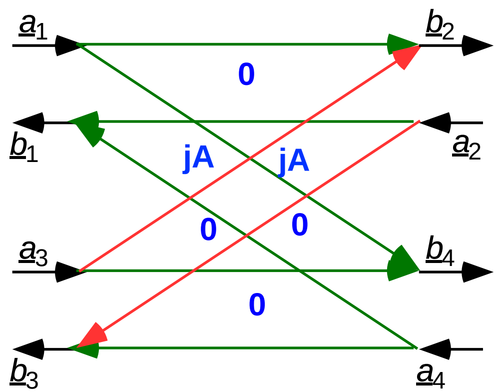
a1
A
b2
1/√2
1/√2
b1
0
a2
j/√2
j/√2
a3
A
b4
1/√2
1/√2
b3
0
a4



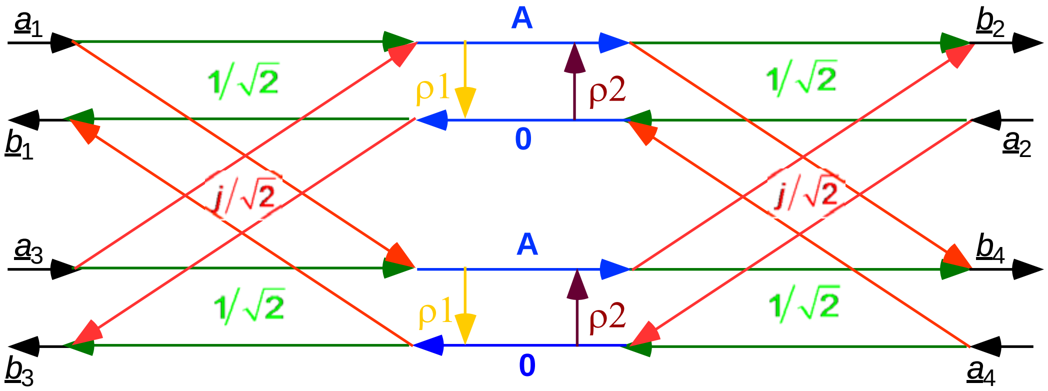
a1
b2
b1
a2
a3
b4
b3
a4



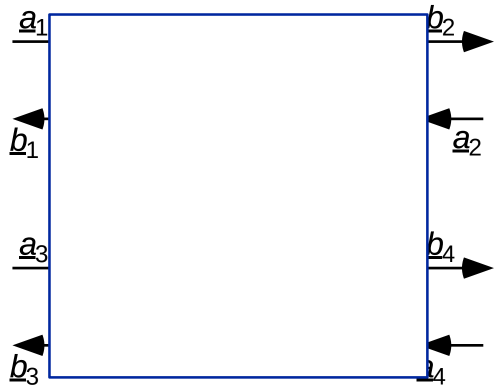
$\underline{a}_1$
$\underline{b}_1$
$\underline{a}_3$
$\underline{b}_3$
$\underline{b}_2$
$\underline{a}_2$
$\underline{b}_4$
$\underline{a}_4$
A
0
A
0
$1/\sqrt{2}$
$j/\sqrt{2}$
$1/\sqrt{2}$
$1/\sqrt{2}$
$j/\sqrt{2}$
$1/\sqrt{2}$



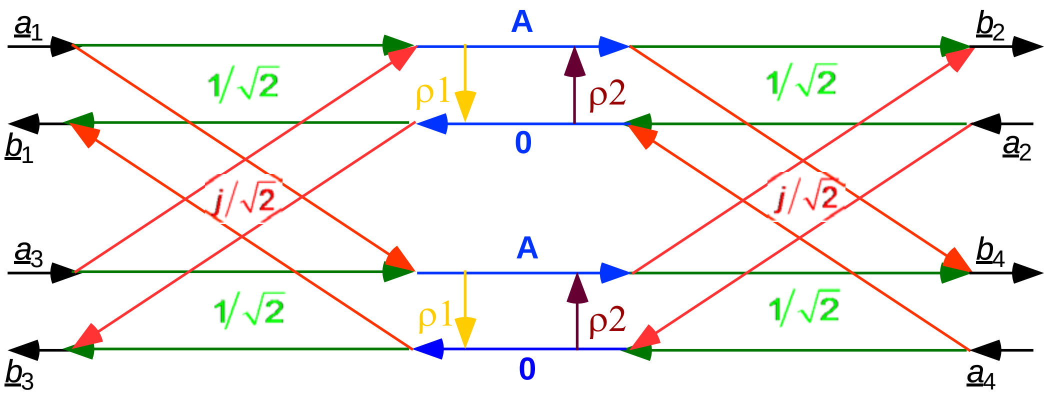
$\underline{a}_1$
$\underline{b}_1$
$\underline{a}_3$
$\underline{b}_3$
$\underline{b}_2$
$\underline{a}_2$
$\underline{b}_4$
$\underline{a}_4$
0
jA
jA
0
0
0

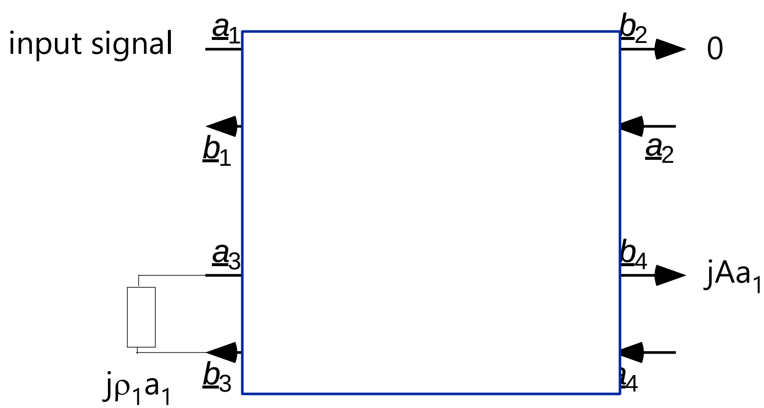


a1
b1
a3
b3
A
0
A
0
ρ1
ρ2
ρ1
ρ2
1/√2
1/√2
1/√2
1/√2
j/√2
j/√2
b2
a2
b4
a4



a1
b1
a3
b3
b2
a2
b4


a1
b2
A
1/√2
ρ1
ρ2
1/√2
b1
0
a2
j/√2
j/√2
a3
A
b4
1/√2
ρ1
ρ2
1/√2
b3
0
a4


input signal
a1
b2
0
b1
a2
a3
b4
jAa1
jρ1a1
b3
a4